Chapter 9

# Electromagnetic induction

***Why it is important to understand:*** **Electromagnetic induction**

**Electromagnetic induction is the production of a potential difference (voltage) across a conductor when it is exposed to a varying magnetic field. Michael Faraday is generally credited with the discovery of induction in the 1830s. Faraday's law of induction is a basic law of electromagnetism that predicts how a magnetic field will interact with an electric circuit to produce an electromotive force (e.m.f.). It is the fundamental operating principle of transformers, inductors and many types of electrical motors, generators and solenoids. A.c. generators use Faraday's law to produce rotation and thus convert electrical and magnetic energy into rotational kinetic energy. This idea can be used to run all kinds of motors. Probably one of the greatest inventions of all time is the transformer. Alternating current from the primary coil moves quickly back and forth across the secondary coil. The moving magnetic field caused by the changing field (flux) induces a current in the secondary coil. This chapter explains electromagnetic induction, Faraday's laws, Lenz's law and Fleming's rule and develops various calculations to help understanding of the concepts.**

**At the end of this chapter you should be able to:**

- understand how an e.m.f. may be induced in a conductor
- state Faraday's laws of electromagnetic induction
- state Lenz's law
- use Fleming's right-hand rule for relative directions
- appreciate that the induced e.m.f., $E = Blv$ or $E = Blv \sin\theta$
- calculate induced e.m.f. given $B$, $l$, $v$ and $\theta$ and determine relative directions
- understand and perform calculations on rotation of a loop in a magnetic field
- define inductance $L$ and state its unit
- define mutual inductance
- appreciate that e.m.f. $E = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}$
- calculate induced e.m.f. given $N$, $t$, $L$, change of flux or change of current
- appreciate factors which affect the inductance of an inductor
- draw the circuit diagram symbols for inductors
- calculate the energy stored in an inductor using $W = \frac{1}{2}LI^2$ joules

- calculate inductance $L$ of a coil, given $L = \frac{N\Phi}{I}$ and $L = \frac{N^2}{S}$
- calculate mutual inductance using $E_2 = -M\frac{dI_1}{dt}$ and $M = \frac{N_1 N_2}{S}$

## 9.1 Introduction to electromagnetic induction

When a conductor is moved across a magnetic field so as to cut through the lines of force (or flux), an electromotive force (e.m.f.) is produced in the conductor. If the conductor forms part of a closed circuit then the e.m.f. produced causes an electric current to flow round the circuit. Hence an e.m.f. (and thus current) is 'induced' in the conductor as a result of its movement across the magnetic field. This effect is known as **'electromagnetic induction'**.

Figure 9.1(a) shows a coil of wire connected to a centre-zero galvanometer, which is a sensitive ammeter with the zero-current position in the centre of the scale.

(a) When the magnet is moved at constant speed towards the coil (Figure 9.1(a)), a deflection is noted on the galvanometer showing that a current has been produced in the coil.

(b) When the magnet is moved at the same speed as in (a) but away from the coil the same deflection is noted but is in the opposite direction (see Figure 9.1(b)).

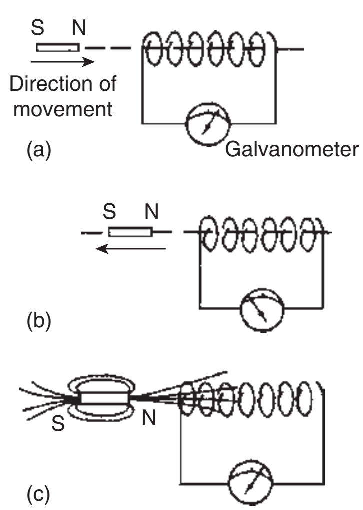


**Figure 9.1**

(c) When the magnet is held stationary, even within the coil, no deflection is recorded.

(d) When the coil is moved at the same speed as in (a) and the magnet held stationary the same galvanometer deflection is noted.

(e) When the relative speed is, say, doubled, the galvanometer deflection is doubled.

(f) When a stronger magnet is used, a greater galvanometer deflection is noted.

(g) When the number of turns of wire of the coil is increased, a greater galvanometer deflection is noted.

Figure 9.1(c) shows the magnetic field associated with the magnet. As the magnet is moved towards the coil, the magnetic flux of the magnet moves across, or cuts, the coil. **It is the relative movement of the magnetic flux and the coil that causes an e.m.f., and thus current, to be induced in the coil.** This effect is known as electromagnetic induction. The laws of electromagnetic induction stated in Section 9.2 evolved from experiments such as those described above.

## 9.2 Laws of electromagnetic induction

**Faraday's* laws of electromagnetic induction state:**

*(i) An induced e.m.f. is set up whenever the magnetic field linking that circuit changes.*

*(ii) The magnitude of the induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit.*

**Lenz's* law states:**

*The direction of an induced e.m.f. is always such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that e.m.f.*

*Who were **Faraday** and **Lenz**? Go to www.routledge.com/cw/bird

An alternative method to Lenz's law of determining relative directions is given by **Fleming's*** **Right-hand rule** (often called the geneRator rule) which states:

*Let the thumb, first finger and second finger of the right hand be extended such that they are all at right-angles to each other (as shown in Figure 9.2).*

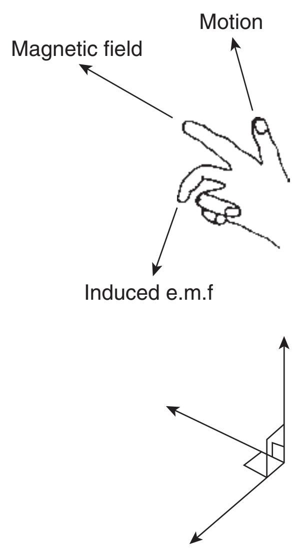


**Figure 9.2**

*If the first finger points in the direction of the magnetic field, the thumb points in the direction of motion of the conductor relative to the magnetic field, then the second finger will point in the direction of the induced e.m.f.*

Summarizing:

First finger – Field

ThuMb – Motion

SEcond finger – E.m.f.

In a generator, conductors forming an electric circuit are made to move through a magnetic field. By Faraday's law an e.m.f. is induced in the conductors and thus a source of e.m.f. is created. A generator converts mechanical energy into electrical energy. (The action of a simple a.c. generator is described in Chapter 14.) The induced e.m.f. $E$ set up between the ends of the conductor shown in Figure 9.3 is given by:

$$E = Blv \text{ volts}$$

where $B$, the flux density, is measured in teslas, $l$, the length of conductor in the magnetic field, is measured

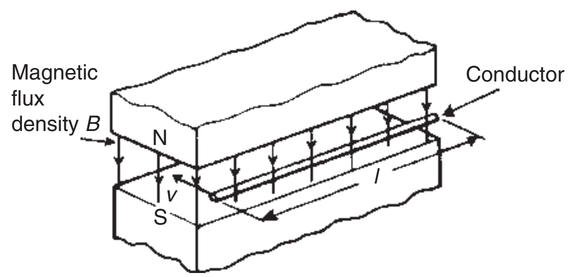


**Figure 9.3**

in metres, and $v$, the conductor velocity, is measured in metres per second.

If the conductor moves at an angle $\theta°$ to the magnetic field (instead of at 90°, as assumed above) then

$$E = Blv \sin\theta \text{ volts}$$

**Problem 1.** A conductor 300 mm long moves at a uniform speed of 4 m/s at right-angles to a uniform magnetic field of flux density 1.25 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 20 Ω resistance.

When a conductor moves in a magnetic field it will have an e.m.f. induced in it, but this e.m.f. can only produce a current if there is a closed circuit.

$$\text{Induced e.m.f. } E = Blv = (1.25)\left(\frac{300}{1000}\right)(4) = 1.5\,\text{V}$$

(a) If the ends of the conductor are open-circuited **no current will flow** even though 1.5 V has been induced.

(b) From Ohm's law, $I = \frac{E}{R} = \frac{1.5}{20} = \mathbf{0.075\,A}$ or **75 mA**

**Problem 2.** At what velocity must a conductor 75 mm long cut a magnetic field of flux density 0.6 T if an e.m.f. of 9 V is to be induced in it? Assume the conductor, the field and the direction of motion are mutually perpendicular.

Induced e.m.f. $E = Blv$, hence velocity $v = \frac{E}{Bl}$

$$\text{Hence} \quad v = \frac{9}{(0.6)(75\times10^{-3})} = \frac{9\times10^3}{0.6\times75} = \mathbf{200\,m/s}$$

*Who was **Fleming**? Go to www.routledge.com/cw/bird

**Problem 3.** A conductor moves with a velocity of 15 m/s at an angle of (a) 90°, (b) 60° and (c) 30° to a magnetic field produced between two square-faced poles of side length 2 cm. If the flux leaving a pole face is 5 μWb, find the magnitude of the induced e.m.f. in each case.

$v = 15$ m/s; length of conductor in magnetic field, $l = 2\,\text{cm} = 0.02\,\text{m}$; $A = 2 \times 2\,\text{cm}^2 = 4 \times 10^{-4}\,\text{m}^2$, $\Phi = 5 \times 10^{-6}\,\text{Wb}$

(a) $$E_{90} = Blv \sin 90° = \left(\frac{\Phi}{A}\right) lv \sin 90° = \frac{(5 \times 10^{-6})}{(4 \times 10^{-4})}(0.02)(15)(1) = \mathbf{3.75\ mV}$$

(b) $$E_{60} = Blv \sin 60° = E_{90} \sin 60° = 3.75 \sin 60° = \mathbf{3.25\ mV}$$

(c) $$E_{30} = Blv \sin 30° = E_{90} \sin 30° = 3.75 \sin 30° = \mathbf{1.875\ mV}$$

**Problem 4.** The wing span of a metal aeroplane is 36 m. If the aeroplane is flying at 400 km/h, determine the e.m.f. induced between its wing tips. Assume the vertical component of the earth's magnetic field is 40 μT.

Induced e.m.f. across wing tips, $E = Blv$

$B = 40\,\mu\text{T} = 40 \times 10^{-6}\,\text{T}$; $l = 36$ m

$$v = 400\frac{\text{km}}{\text{h}} \times 1000\frac{\text{m}}{\text{km}} \times \frac{1\,\text{h}}{60 \times 60\,\text{s}} = \frac{(400)(1000)}{3600} = \frac{4000}{36}\,\text{m/s}$$

Hence $$E = Blv = (40 \times 10^{-6})(36)\left(\frac{4000}{36}\right) = \mathbf{0.16\ V}$$

**Problem 5.** The diagram shown in Figure 9.4 represents the generation of e.m.f.s. Determine (i) the direction in which the conductor has to be moved in Figure 9.4(a), (ii) the direction of the induced e.m.f. in Figure 9.4(b), (iii) the polarity of the magnetic system in Figure 9.4(c).

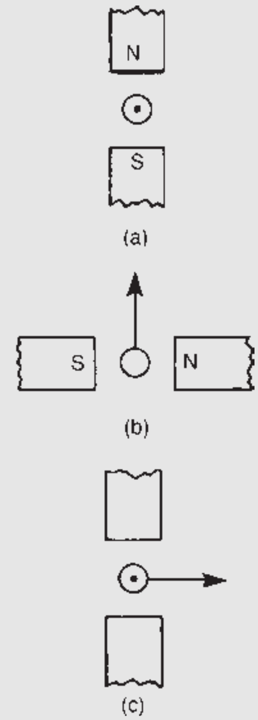


**Figure 9.4**

The direction of the e.m.f., and thus the current due to the e.m.f., may be obtained by either Lenz's law or Fleming's Right-hand rule (i.e. GeneRator rule).

(i) Using Lenz's law: the field due to the magnet and the field due to the current-carrying conductor are shown in Figure 9.5(a) and are seen to reinforce to the left of the conductor. Hence the force on the conductor is to the right. However, Lenz's law states that the direction of the induced e.m.f. is always such as to oppose the effect producing it. **Thus the conductor will have to be moved to the left.**

(ii) Using Fleming's right-hand rule:
First finger – Field, i.e. $N \rightarrow S$, or right to left;
ThuMb – Motion, i.e. upwards;
SEcond finger – E.m.f., i.e. **towards the viewer or out of the paper**, as shown in Figure 9.5(b)

(iii) The polarity of the magnetic system of Figure 9.4(c) is shown in Figure 9.5(c) and is obtained using Fleming's right-hand rule.

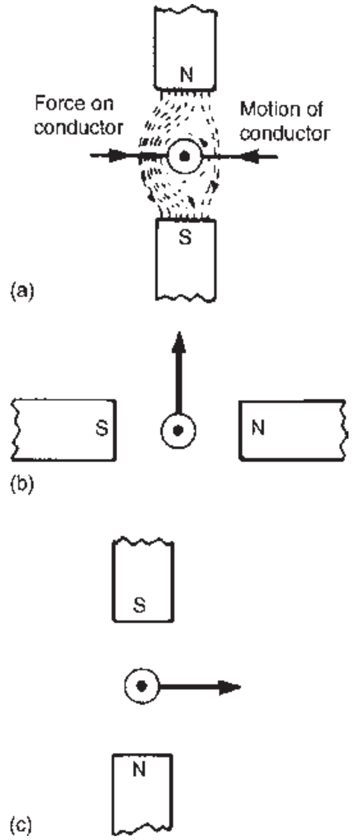


Figure 9.5

**Now try the following Practice Exercise**

**Practice Exercise 26 Induced e.m.f. (Answers on page 745)**

1. A conductor of length 15 cm is moved at 750 mm/s at right-angles to a uniform flux density of 1.2 T. Determine the e.m.f. induced in the conductor.
2. Find the speed that a conductor of length 120 mm must be moved at right-angles to a magnetic field of flux density 0.6 T to induce in it an e.m.f. of 1.8 V
3. A 25 cm long conductor moves at a uniform speed of 8 m/s through a uniform magnetic field of flux density 1.2 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 15 ohms resistance.
4. A car is travelling at 80 km/h. Assuming the back axle of the car is 1.76 m in length and the vertical component of the earth's magnetic field is 40 μT, find the e.m.f. generated in the axle due to motion.
5. A conductor moves with a velocity of 20 m/s at an angle of (a) 90° (b) 45° (c) 30°, to a magnetic field produced between two square-faced poles of side length 2.5 cm. If the flux on the pole face is 60 mWb, find the magnitude of the induced e.m.f. in each case.
6. A conductor 400 mm long is moved at 70° to a 0.85 T magnetic field. If it has a velocity of 115 km/h, calculate (a) the induced voltage, and (b) force acting on the conductor if connected to an 8 Ω resistor.

## 9.3 Rotation of a loop in a magnetic field

Figure 9.6 shows a view of a looped conductor whose sides are moving across a magnetic field.

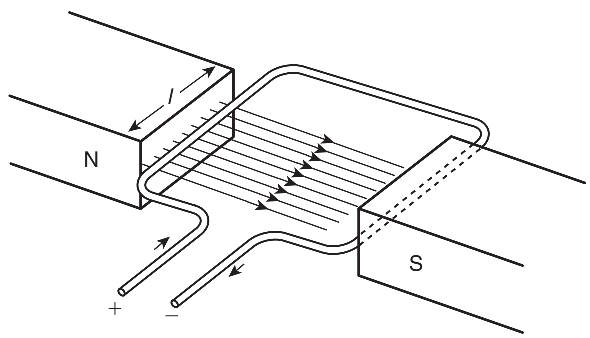


Figure 9.6

The left-hand side is moving in an upward direction (check using Fleming's right-hand rule), with length $l$ cutting the lines of flux which are travelling from left to right. By definition, the induced e.m.f. will be equal to $Blv \sin\theta$ and flowing into the page.

The right-hand side is moving in a downward direction (again, check using Fleming's right-hand rule), with length $l$ cutting the same lines of flux as above. The induced e.m.f. will also be equal to $Blv \sin\theta$ but flowing out of the page.

Therefore the total e.m.f. for the loop conductor $= 2Blv \sin\theta$

Now consider a coil made up of a number of turns $N$.

The total e.m.f. $E$ for the loop conductor is now given by:

$$E = 2N\,Blv\,\sin\theta$$

**Problem 6.** A rectangular coil of sides 12 cm and 8 cm is rotated in a magnetic field of flux density 1.4 T, the longer side of the coil actually cutting this flux. The coil is made up of 80 turns and rotates at 1200 rev/min.

(a) Calculate the maximum generated e.m.f.

(b) If the coil generates 90 V, at what speed will the coil rotate?

(a) Generated e.m.f. $E = 2\,N\,Blv\sin\theta$

where number of turns, $N = 80$, flux density, $B = 1.4$ T,

length of conductor in magnetic field, $l = 12\text{ cm} = 0.12\text{ m}$,

$$\text{velocity, } v = \omega r = \left(\frac{1200}{60} \times 2\pi \text{ rad/s}\right)\left(\frac{0.08}{2}\text{ m}\right)$$

$$= 1.6\pi \text{ m/s},$$

and for maximum e.m.f. induced, $\theta = 90°$, from which, $\sin\theta = 1$

Hence, **maximum e.m.f. induced,**

$$\boldsymbol{E} = 2\,N\,Blv\sin\theta$$

$$= 2 \times 80 \times 1.4 \times 0.12 \times 1.6\pi \times 1 = \mathbf{135.1\ volts}$$

(b) Since $E = 2\,N\,Blv\sin\theta$

then $90 = 2 \times 80 \times 1.4 \times 0.12 \times v \times 1$

from which, $$v = \frac{90}{2 \times 80 \times 1.4 \times 0.12} = 3.348\text{ m/s}$$

$v = \omega r$ hence, angular velocity,

$$\omega = \frac{v}{r} = \frac{3.348}{\frac{0.08}{2}} = 83.7\text{ rad/s}$$

$$\textbf{Speed of coil in rev/min} = \frac{83.7 \times 60}{2\pi} = \mathbf{799\ rev/min}$$

An **alternative method** of determining (b) is by **direct proportion**.

Since $E = 2N\,Blv\sin\theta$, then with $N$, $B$, $l$ and $\theta$ being constant, $\boldsymbol{E \propto v}$

If from (a), 135.1 V is produced by a speed of 1200 rev/min,

then 1 V would be produced by a speed of $\dfrac{1200}{135.1}$

$= 8.88$ rev/min

Hence, 90 V would be produced by a speed of $90 \times 8.88 =$ **799 rev/min**

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**Now try the following Practice Exercise**

**Practice Exercise 27 Induced e.m.f. in a coil (Answers on page 745)**

1. A rectangular coil of sides 8 cm by 6 cm is rotating in a magnetic field such that the longer sides cut the magnetic field. Calculate the maximum generated e.m.f. if there are 60 turns on the coil, the flux density is 1.6 T and the coil rotates at 1500 rev/min.
2. A generating coil on a former 100 mm long has 120 turns and rotates in a 1.4 T magnetic field. Calculate the maximum e.m.f. generated if the coil, having a diameter of 60 mm, rotates at 450 rev/min.
3. If the coils in Problems 1 and 2 generate 60 V, calculate (a) the new speed for each coil, and (b) the flux density required if the speed is unchanged.

## 9.4 Inductance

**Inductance** is the name given to the property of a circuit whereby there is an e.m.f. induced into the circuit by the change of flux linkages produced by a current change.

When the e.m.f. is induced in the same circuit as that in which the current is changing, the property is called **self inductance**, $\boldsymbol{L}$.

When the e.m.f. is induced in a circuit by a change of flux due to current changing in an adjacent circuit, the property is called **mutual inductance**, $\boldsymbol{M}$.

The unit of inductance is the **henry,*** **H**.

*A circuit has an inductance of one henry when an e.m.f. of one volt is induced in it by a current changing at the rate of one ampere per second.*

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*Who was **Henry**? Go to www.routledge.com/cw/bird

Induced e.m.f. in a coil of $N$ turns,

$$\mathbf{E = -N\frac{d\Phi}{dt}\ volts}$$

where d$\Phi$ is the change in flux in Webers, and d$t$ is the time taken for the flux to change in seconds (i.e. d$\Phi$/d$t$ is the rate of change of flux).

Induced e.m.f. in a coil of inductance $L$ henrys,

$$\mathbf{E = -L\frac{dI}{dt}\ volts}$$

where d$I$ is the change in current in amperes and d$t$ is the time taken for the current to change in seconds (i.e. d$I$/d$t$ is the rate of change of current). The minus sign in each of the above two equations remind us of its direction (given by Lenz's law).

**Problem 7.** Determine the e.m.f. induced in a coil of 200 turns when there is a change of flux of 25 mWb linking with it in 50 ms.

$$\text{Induced e.m.f. } E = -N\frac{d\Phi}{dt} = -(200)\left(\frac{25\times10^{-3}}{50\times10^{-3}}\right)$$

$$= \mathbf{-100\ volts}$$

**Problem 8.** A flux of 400 μWb passing through a 150-turn coil is reversed in 40 ms. Find the average e.m.f. induced.

Since the flux reverses, the flux changes from +400 μWb to −400 μWb, a total change of flux of 800 μWb

$$\text{Induced e.m.f. } E = -N\frac{d\Phi}{dt} = -(150)\left(\frac{800\times10^{-6}}{40\times10^{-3}}\right)$$

$$= -\left(\frac{150\times800\times10^{3}}{40\times10^{6}}\right)$$

Hence **the average e.m.f. induced $E = -3$ volts**

**Problem 9.** Calculate the e.m.f. induced in a coil of inductance 12 H by a current changing at the rate of 4 A/s.

$$\text{Induced e.m.f. } E = -L\frac{dI}{dt} = -(12)(4) = \mathbf{-48\ volts}$$

**Problem 10.** An e.m.f. of 1.5 kV is induced in a coil when a current of 4 A collapses uniformly to zero in 8 ms. Determine the inductance of the coil.

Change in current, d$I$ = (4 − 0) = 4 A;
d$t$ = 8 ms = $8\times10^{-3}$ s;

$$\frac{dI}{dt} = \frac{4}{8\times10^{-3}} = \frac{4000}{8} = 500\ \text{A/s};$$

$$E = 1.5\,\text{kV} = 1500\,\text{V}$$

$$\text{Since } |E| = L\left(\frac{dI}{dt}\right)$$

$$\text{inductance, } L = \frac{|E|}{(dI/dt)} = \frac{1500}{500} = \mathbf{3\ H}$$

(Note that $|E|$ means the 'magnitude of $E$', which disregards the minus sign.)

**Now try the following Practice Exercise**

**Practice Exercise 28 Inductance (Answers on page 745)**

1. Find the e.m.f. induced in a coil of 200 turns when there is a change of flux of 30 mWb linking with it in 40 ms.
2. An e.m.f. of 25 V is induced in a coil of 300 turns when the flux linking with it changes by 12 mWb. Find the time, in milliseconds, in which the flux makes the change.
3. An ignition coil having 10 000 turns has an e.m.f. of 8 kV induced in it. What rate of change of flux is required for this to happen?
4. A flux of 0.35 mWb passing through a 125-turn coil is reversed in 25 ms. Find the magnitude of the average e.m.f. induced.

## 9.5 Inductors

A component called an inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire.
Factors which affect the inductance of an inductor include:

(i) the number of turns of wire – the more turns the higher the inductance

(ii) the cross-sectional area of the coil of wire – the greater the cross-sectional area the higher the inductance

(iii) the presence of a magnetic core – when the coil is wound on an iron core the same current sets up a more concentrated magnetic field and the inductance is increased

(iv) the way the turns are arranged – a short, thick coil of wire has a higher inductance than a long, thin one.

Two examples of practical inductors are shown in Figure 9.7, and the standard electrical circuit diagram symbols for air-cored and iron-cored inductors are shown in Figure 9.8.

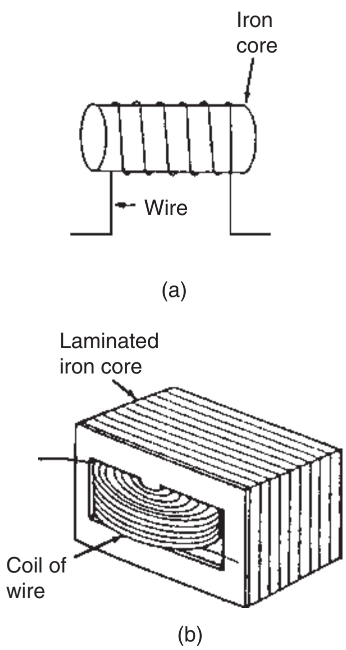


Figure 9.7

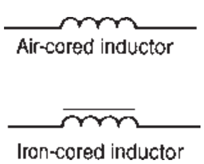


Figure 9.8

An iron-cored inductor is often called a **choke** since, when used in a.c. circuits, it has a choking effect, limiting the current flowing through it. Inductance is often undesirable in a circuit. To reduce inductance to a minimum the wire may be bent back on itself, as shown in Figure 9.9, so that the magnetizing effect of one conductor is neutralized by that of the adjacent conductor. The wire may be coiled around an insulator, as shown, without increasing the inductance. Standard resistors may be non-inductively wound in this manner.

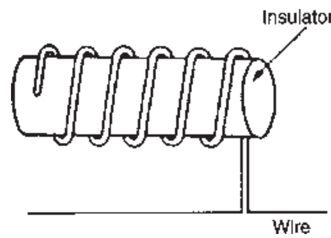


Figure 9.9

## 9.6 Energy stored

An inductor possesses an ability to store energy. The energy stored, $W$, in the magnetic field of an inductor is given by:

$$W = \frac{1}{2}LI^2 \text{ joules}$$

**Problem 11.** An 8 H inductor has a current of 3 A flowing through it. How much energy is stored in the magnetic field of the inductor?

Energy stored, $W = \frac{1}{2}LI^2 = \frac{1}{2}(8)(3)^2 = \mathbf{36\ joules}$

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**Now try the following Practice Exercise**

### Practice Exercise 29 Energy stored (Answers on page 745)

1. Calculate the value of the energy stored when a current of 30 mA is flowing in a coil of inductance 400 mH.

2. The energy stored in the magnetic field of an inductor is 80 J when the current flowing in the inductor is 2 A. Calculate the inductance of the coil.

## 9.7 Inductance of a coil

If a current changing from 0 to $I$ amperes produces a flux change from 0 to $\Phi$ Webers, then $dI = I$ and $d\Phi = \Phi$. Then, from Section 9.4, induced e.m.f. $E = N\Phi/t = LI/t$, from which

**inductance of coil,** $L = \dfrac{N\Phi}{I}$ **henrys**

$$\text{Since } E = -L\frac{dI}{dt} = -N\frac{d\Phi}{dt} \text{ then } L = N\frac{d\Phi}{dt}\left(\frac{dt}{dI}\right)$$

$$\text{i.e.} \quad L = N\frac{d\Phi}{dI}$$

From Chapter 7, mmf $= \Phi S$ from which, $\Phi = \dfrac{\text{mmf}}{S}$

$$\text{Substituting into} \quad L = N\frac{d\Phi}{dI}$$

$$\text{gives} \quad L = N\frac{d}{dI}\left(\frac{\text{mmf}}{S}\right)$$

$$\text{i.e.} \quad L = \frac{N}{S}\frac{d(NI)}{dI} \quad \text{since mmf} = NI$$

$$\text{i.e.} \quad L = \frac{N^2}{S}\frac{dI}{dI} \quad \text{and since } \frac{dI}{dI} = 1,$$

$$L = \frac{N^2}{S} \text{ henrys}$$

**Problem 12.** Calculate the coil inductance when a current of 4 A in a coil of 800 turns produces a flux of 5 mWb linking with the coil.

For a coil, inductance $L = \dfrac{N\Phi}{I}$

$$= \frac{(800)(5 \times 10^{-3})}{4} = \mathbf{1\,H}$$

**Problem 13.** A flux of 25 mWb links with a 1500 turn coil when a current of 3 A passes through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field and (c) the average e.m.f. induced if the current falls to zero in 150 ms.

(a) **Inductance,** $L = \dfrac{N\Phi}{I} = \dfrac{(1500)(25 \times 10^{-3})}{3} = \mathbf{12.5\,H}$

(b) **Energy stored,** $W = \frac{1}{2}LI^2 = \frac{1}{2}(12.5)(3)^2 = \mathbf{56.25\,J}$

(c) **Induced e.m.f.,** $E = -L\dfrac{dI}{dt}$

$$= -(12.5)\left(\frac{3-0}{150 \times 10^{-3}}\right) = \mathbf{-250\,V}$$

(Alternatively, $E = -N\left(\dfrac{d\Phi}{dt}\right)$

$$= -(1500)\left(\frac{25 \times 10^{-3}}{150 \times 10^{-3}}\right) = \mathbf{-250\,V}$$

since if the current falls to zero so does the flux.)

**Problem 14.** A 750-turn coil of inductance 3 H carries a current of 2 A. Calculate the flux linking the coil and the e.m.f. induced in the coil when the current collapses to zero in 20 ms.

Coil inductance, $L = \dfrac{N\Phi}{I}$ from which,

$$\text{flux } \Phi = \frac{LI}{N} = \frac{(3)(2)}{750} = 8 \times 10^{-3} = \mathbf{8\,mWb}$$

$$\text{Induced e.m.f.} \quad E = -L\left(\frac{dI}{dt}\right) = -3\left(\frac{2-0}{20 \times 10^{-3}}\right) = \mathbf{-300\,V}$$

(Alternatively, $E = -N\dfrac{d\Phi}{dt} = -(750)\left(\dfrac{8 \times 10^{-3}}{20 \times 10^{-3}}\right) = \mathbf{-300\,V}$)

**Problem 15.** A silicon iron ring is wound with 800 turns, the ring having a mean diameter of 120 mm and a cross-sectional area of 400 mm$^2$. If when carrying a current of 0.5 A the relative permeability is found to be 3000, calculate (a) the self inductance of the coil, (b) the induced e.m.f. if the current is reduced to zero in 80 ms.

The ring is shown sketched in Figure 9.10.

(a) Inductance, $L = \dfrac{N^2}{S}$ and from Chapter 7,

reluctance, $S = \dfrac{l}{\mu_0 \mu_r A}$

$$\text{i.e.} \quad S = \frac{\pi \times 120 \times 10^{-3}}{4\pi \times 10^{-7} \times 3000 \times 400 \times 10^{-6}}$$

$$= 250 \times 10^3 \text{ A/Wb}$$

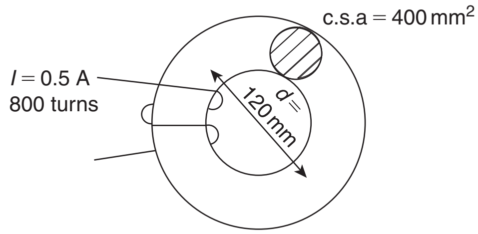


Figure 9.10

Hence, **self inductance,** $L = \frac{N^2}{S} = \frac{800^2}{250 \times 10^3}$

$= \mathbf{2.56\ H}$

(b) **Induced e.m.f.,** $E = -L\frac{dI}{dt}$

$= -(2.56)\frac{(0.5-0)}{80 \times 10^{-3}}$

$= \mathbf{-16\ V}$

**Now try the following Practice Exercise**

**Practice Exercise 30 Inductance of a coil (Answers on page 745)**

1. A flux of 30 mWb links with a 1200-turn coil when a current of 5 A is passing through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field and (c) the average e.m.f. induced if the current is reduced to zero in 0.20 s
2. An e.m.f. of 2 kV is induced in a coil when a current of 5 A collapses uniformly to zero in 10 ms. Determine the inductance of the coil.
3. An average e.m.f. of 60 V is induced in a coil of inductance 160 mH when a current of 7.5 A is reversed. Calculate the time taken for the current to reverse.
4. A coil of 2500 turns has a flux of 10 mWb linking with it when carrying a current of 2 A. Calculate the coil inductance and the e.m.f. induced in the coil when the current collapses to zero in 20 ms.
5. A coil is wound with 600 turns and has a self inductance of 2.5 H. What current must flow to set up a flux of 20 mWb?
6. When a current of 2 A flows in a coil, the flux linking with the coil is 80 μWb. If the coil inductance is 0.5 H, calculate the number of turns of the coil.
7. A steady current of 5 A when flowing in a coil of 1000 turns produces a magnetic flux of 500 μWb. Calculate the inductance of the coil. The current of 5 A is then reversed in 12.5 ms. Calculate the e.m.f. induced in the coil.
8. An iron ring has a cross-sectional area of 500 mm$^2$ and a mean length of 300 mm. It is wound with 100 turns and its relative permeability is 1600. Calculate (a) the current required to set up a flux of 500 μWb in the coil, (b) the inductance of the system and (c) the induced e.m.f. if the field collapses in 1 ms.

## 9.8 Mutual inductance

Mutually induced e.m.f. in the second coil,

$$E_2 = -M\frac{dI_1}{dt} \text{ volts}$$

where $M$ is the **mutual inductance** between two coils, in henrys, and $dI_1/dt$ is the rate of change of current in the first coil.

The phenomenon of mutual inductance is used in **transformers** (see Chapter 20, page 280). Mutual inductance is developed further in Chapter 43 on magnetically coupled circuits (see page 656).

### Another expression for *M*

Let an iron ring have two coils, A and B, wound on it. If the fluxes $\Phi_1$ and $\Phi_2$ are produced from currents $I_1$ and $I_2$ in coils A and B respectively, then the reluctance could be expressed as:

$$S = \frac{I_1 N_1}{\Phi_1} = \frac{I_2 N_2}{\Phi_2}$$

If the flux in coils A and B are the same and produced from the current $I_1$ in coil A only, assuming 100% coupling, then the mutual inductance can be expressed as:

$$M = \frac{N_2 \Phi_1}{I_1}$$

Multiplying by $\left(\frac{N_1}{N_1}\right)$ gives: $M = \frac{N_2 \Phi_1 N_1}{I_1 N_1}$

However, $S = \frac{I_1 N_1}{\Phi_1}$

Thus, mutual inductance, $\mathbf{M = \frac{N_1 N_2}{S}}$

**Problem 16.** Calculate the mutual inductance between two coils when a current changing at 200 A/s in one coil induces an e.m.f. of 1.5 V in the other.

Induced e.m.f. $|E_2| = M\frac{dI_1}{dt}$, i.e. $1.5 = M(200)$

Thus **mutual inductance,** $M = \frac{1.5}{200}$

$= \mathbf{0.0075\ H}$ or $\mathbf{7.5\ mH}$

**Problem 17.** The mutual inductance between two coils is 18 mH. Calculate the steady rate of change of current in one coil to induce an e.m.f. of 0.72 V in the other.

Induced e.m.f., $|E_2| = M\frac{dI_1}{dt}$

Hence rate of change of current, $\frac{dI_1}{dt} = \frac{|E_2|}{M}$

$= \frac{0.72}{0.018} = \mathbf{40\ A/s}$

**Problem 18.** Two coils have a mutual inductance of 0.2 H. If the current in one coil is changed from 10 A to 4 A in 10 ms, calculate (a) the average induced e.m.f. in the second coil, (b) the change of flux linked with the second coil if it is wound with 500 turns.

(a) Induced e.m.f. $E_2 = -M\frac{dI_1}{dt}$

$= -(0.2)\left(\frac{10-4}{10 \times 10^{-3}}\right)$

$= \mathbf{-120\ V}$

(b) Induced e.m.f. $|E_2| = N\frac{d\Phi}{dt}$, hence $d\Phi = \frac{|E_2|dt}{N}$

Thus the change of flux, $d\Phi = \frac{120(10 \times 10^{-3})}{500}$

$= \mathbf{2.4\ mWb}$

**Problem 19.** In the device shown in Figure 9.11, when the current in the primary coil of 1000 turns increases linearly from 1 A to 6 A in 200 ms, an e.m.f. of 15 V is induced into the secondary coil of 480 turns, which is left open circuited. Determine (a) the mutual inductance of the two coils, (b) the reluctance of the former and (c) the self inductance of the primary coil.

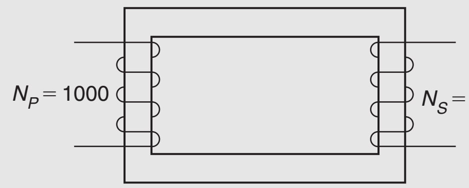


Figure 9.11

(a) $E_S = M\frac{dI_P}{dt}$ from which,

mutual inductance, $\mathbf{M} = \frac{E_S}{\frac{dI_P}{dt}} = \frac{15}{\left(\frac{6-1}{200 \times 10^{-3}}\right)}$

$= \frac{15}{25} = \mathbf{0.60\ H}$

(b) $M = \frac{N_P N_S}{S}$ from which,

reluctance, $S = \frac{N_P N_S}{M} = \frac{(1000)(480)}{0.60}$

$= \mathbf{800\,000\ A/Wb}$ or $\mathbf{800\ kA/Wb}$

(c) Primary self inductance, $L_P = \frac{N_P^2}{S} = \frac{(1000)^2}{800\,000}$

$= \mathbf{1.25\ H}$

**Now try the following Practice Exercise**

### Practice Exercise 31 Mutual inductance (Answers on page 745)

1. The mutual inductance between two coils is 150 mH. Find the magnitude of the e.m.f. induced in one coil when the current in the other is increasing at a rate of 30 A/s.

2. Determine the mutual inductance between two coils when a current changing at 50 A/s in one coil induces an e.m.f. of 80 mV in the other.

3. Two coils have a mutual inductance of 0.75 H. Calculate the magnitude of the e.m.f. induced in one coil when a current of 2.5 A in the other coil is reversed in 15 ms.

4. The mutual inductance between two coils is 240 mH. If the current in one coil changes from 15 A to 6 A in 12 ms, calculate (a) the average e.m.f. induced in the other coil, (b) the change of flux linked with the other coil if it is wound with 400 turns.

5. When the current in the primary coil of 400 turns of a magnetic circuit increases linearly from 10 mA to 35 mA in 100 ms, an e.m.f. of 75 mV is induced into the secondary coil of 240 turns, which is left open circuited. Determine (a) the mutual inductance of the two coils, (b) the reluctance of the former and (c) the self inductance of the secondary coil.

**For fully worked solutions to each of the problems in Practice Exercises 26 to 31 in this chapter, go to the website:**
**www.routledge.com/cw/bird**