Lecture 2: Electrical Circuits

In an electrical circuit resistors may be connected in series, in parallel, or in various combinations of series and parallel connections.

2.1 Series circuits

In any series circuit a current *I* will flow through all parts of the circuit as a result of the potential difference supplied by a battery V_T. Therefore, we say that in a series circuit the current is common throughout that circuit.

When the current flows through each resistor in the circuit, for example, R_1, R_2 and R_3 in Fig. 2.1, there will be a voltage drop across that resistor whose value will be determined by the values of I and R, since from Ohm's law V = I X R. The sum of the individual voltage drops, for example, V_1, V_2 and V_3 in Fig. 2.1, will be equal to the total voltage V_T.

For any series circuit, *I* is common throughout the circuit and

$$V_T = V_1 + V_2 + V_3$$

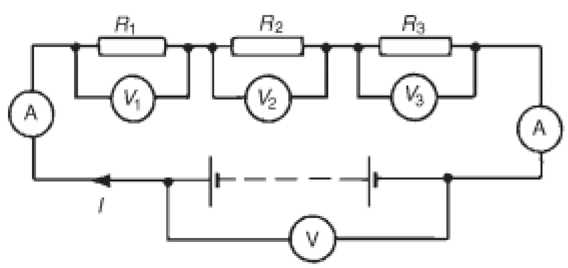

FIGURE 2.1

Figure 2.1 shows three resistors R_1, R_2 and R_3 connected end to end, i.e., in series, with a battery source of V volts. Since the circuit is closed a current I will flow and the p.d. across each resistor may be determined from the voltmeter readings V_1, V_2 and V_3.

In a series circuit;

(a) The current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and

(b) The sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage, V, i.e.

From Ohm's law:

$$V_1 = IR_{1,} V_2 = IR_2\,, V_3 = II_3 \text{ and } V = IR$$

Where R is the total circuit resistance.

$$Since\ V = V_1 + V_2 + V_3$$

$$Then\ IR = IR_1 + IR_2 + IR_3$$

Dividing throughout by I gives,

$$R = R_1 + R_2 + R_3$$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.

Example 2.1

For the circuit shown in Figure 2.2, determine

(a) The battery voltage V,

(b) The total resistance of the circuit, and

(c) The values of resistance of resistors R_1, R_2 and R_3, given that the Potential differences across R_1, R_2 and R_3 are 5V, 2V and 6V respectively.

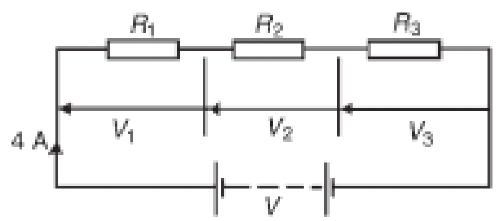

FIGURE 2.2

Solution

$$(a) Battery\ voltage\ V = V_1 + V_2 + V_3$$

$$= 5 + 2 + 6 = 13V$$

$$(b) Total\ circuit\ resistance\ R = \frac{V}{I} = \frac{13}{4} = 3.25\Omega$$

$$(c) Resistance\ R_1 = \frac{V_1}{I} = \frac{5}{4} = 1.25\Omega$$

$$Resistance\ R_2 = \frac{V_2}{I} = \frac{2}{4} = 0.5\Omega$$

$$Resistance\ R_1 = \frac{V_3}{I} = \frac{6}{4} = 1.5\Omega$$

2.2 Parallel networks

In any parallel circuit, as shown in Fig. 2.3, the same voltage acts across all branches of the circuit. The total current will divide when it reaches a resistor junction, part of it flowing in each resistor. The sum of the individual currents, for example, I_1, I_2 and I_3 in Fig. 2.3, will be equal to the total current I_T.

Figure 2.3 below shows three resistors, R_1, R_2 and R_3 connected across each other, i.e., in parallel, across a battery source of V volts.

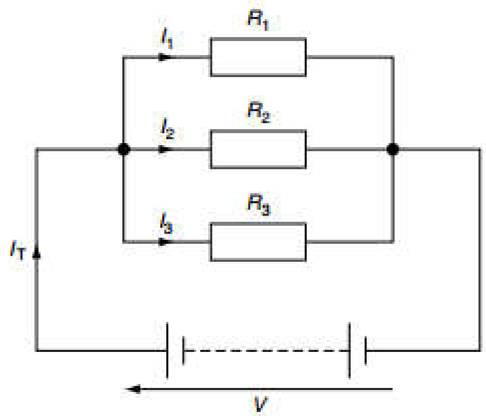

FIGURE 2.3. A parallel circuit.

In a parallel circuit:
(a) The sum of the currents I_1, I_2 and I_3 is equal to the total circuit current, I, i.e. $I = I_1 + I_2 + I_3$, and
(b) The source p.d., V volts, is the same across each of the resistors.
From Ohm's law:

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \text{ and } I = \frac{V}{R}$$

Where R is the total circuit resistance.

$$\text{Since } I = I_1 + I_2 + I_3$$

$$\text{then, } \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Dividing throughout by V gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

This equation must be used when finding the total resistance R of a parallel circuit. For the special case of two resistors in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R = \frac{R_2 R_1}{R_1 + R_2}$$

$$i.e, \frac{Product}{Sum}$$

Exercise 2.2

For the circuit shown in Figure 2.3, determine (a) the reading on the ammeter, and (b) the value of resistor R_2.

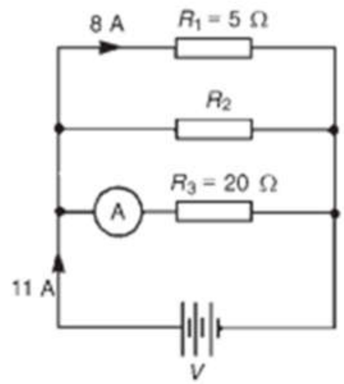

FIGURE 2.3

$$P.d\ across\ R_1\ is\ the\ same\ as\ the\ supply\ voltage\ V.$$

$$Hence\ supply\ voltage, V = 8 \times 5 = 40V$$

$$(a) Reading\ on\ ammeter, I = \frac{V}{R_3} = \frac{40}{20} = 2A$$

$$(b) Current flowing\ through\ R_2 = \frac{V}{I_2} = \frac{40}{1} = 40\Omega$$