LECTURE 8: Three Phase Systems

As mentioned in earlier lectures, generation, transmission and distribution of electricity via the National Grid system is accomplished by three phase alternating currents. The voltage induced by a single coil when rotated in a uniform magnetic field is known as a single-phase voltage. Most consumers are fed by means of a single-phase a.c. supply. Two wires are used, one called the live conductor (usually coloured red) and the other is called the neutral conductor (usually coloured black).

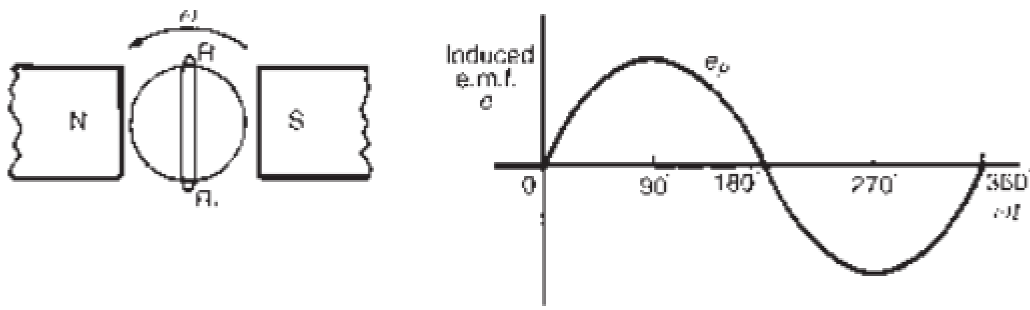


Figure 8.1

8.1 Three Phase Supply

A three-phase supply is generated when three coils are placed 120^0 apart and the whole rotated in a uniform magnetic field.

The result is three independent supplies of equal voltages which are each displaced by 120◦ from each other as shown in the figure 8.2 below:

1. The convention adopted to identify each of the phase voltages is: R-red, Y-yellow, and B-blue.
2. The phase-sequence is given by the sequence in which the conductors pass the point initially taken by the red conductor. The national standard phase sequence is R,Y, B

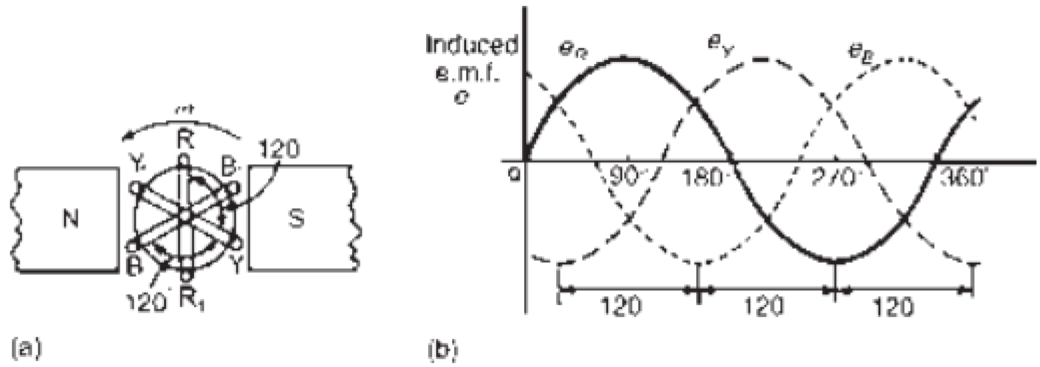


Figure 8.2

A three-phase a.c. supply is carried by three conductors, called 'lines' which are coloured red, yellow and blue. The currents in these conductors are known as line currents (I_L) and the p.d.s between them are known as line voltages (V_L). A fourth conductor, called the neutral (coloured black, and connected through protective devices to earth) is often used with a three-phase supply.

If the three-phase windings shown in **Figure 8.2** are kept independent then six wires are needed to connect a supply source (such as a generator) to a load (such as a motor). To reduce the number of wires it is usual to interconnect the three phases. There are two ways in which this can be done, these being: (a) a **star connection**, and (b) a **delta**, or **mesh, connection**.

Sources of three phase supplies, i.e. alternators, are usually connected in star, whereas three-phase transformer windings, motors and other loads may be connected either in star or delta.

8.2 Star Connection

(i) A **star-connected load** is shown in Figure 8.3, where the three line conductors are each connected to a load and the outlets from the loads are joined together at *N* to form what is termed the **neutral point** or the **star point.**

(ii) The voltages, V_R, V_Y and V_B are called **phase voltages** or line to neutral voltages. Phase voltages are generally denoted by V_p.

The voltages, V_{RY}, V_{YB} and V_{BR} are called **line voltages.**

(iii) From **Figure 8.3** it can be seen that the phase currents (generally denoted by I_p) are equal to their respective line currents I_R, I_Y and I_B, i.e. for a star connection:

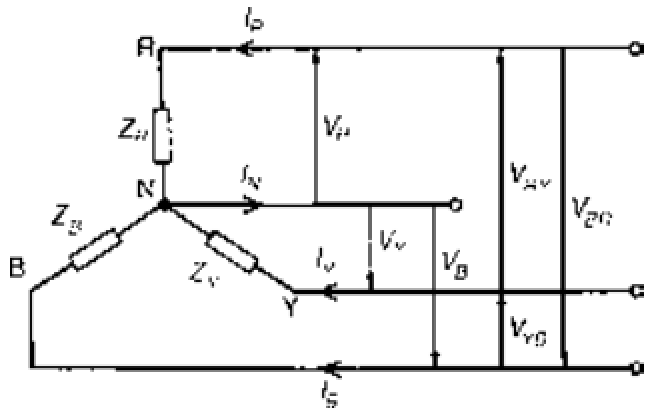


Figure 8.3

(iv) For a balanced system:

$I_R = I_Y = I_B$, $V_R = V_Y = V_B$ $V_{RY} = V_{YB} = V_{BR}$, $Z_R = Z_Y = Z_B$ and the current in the neutral conductor, $I_N = 0$.

When a star-connected system is balanced, then the neutral conductor is unnecessary and is often omitted.

(v)The line voltage, VRY, shown in Figure 8 (a) is given by $\boldsymbol{V_{RY} = V_R - V_Y}$ (V_Y is negative since it is in the opposite direction to V_{RY}).In the phasor diagram of Figure 8.4(b), phasor V_Y is reversed (shown by the broken line) and then added phasorially to V_R (i.e. $\boldsymbol{V_{RY} = V_R + (-V_Y)}$). By trigonometry, or by measurement, $V_{RY} = \sqrt{3}V_R$, i.e. for a balanced star connection:

$$V_L = \sqrt{3}V_p$$

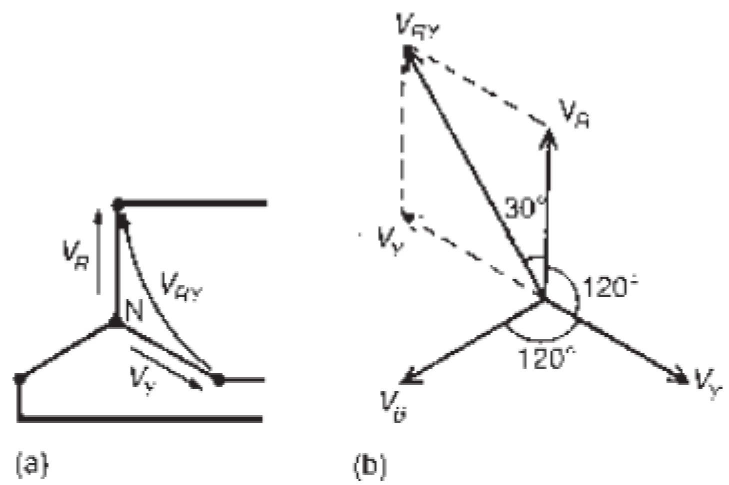


Figure 8.4

The star connection of the three phases of a supply, together with a neutral conductor, allows the use of two voltages – the phase voltage and the line voltage. A four-wire system is also used when the load is not balanced. The standard electricity supply to consumers in Kenya is 415/240V, 50Hz, three phase, four-wire alternating current.

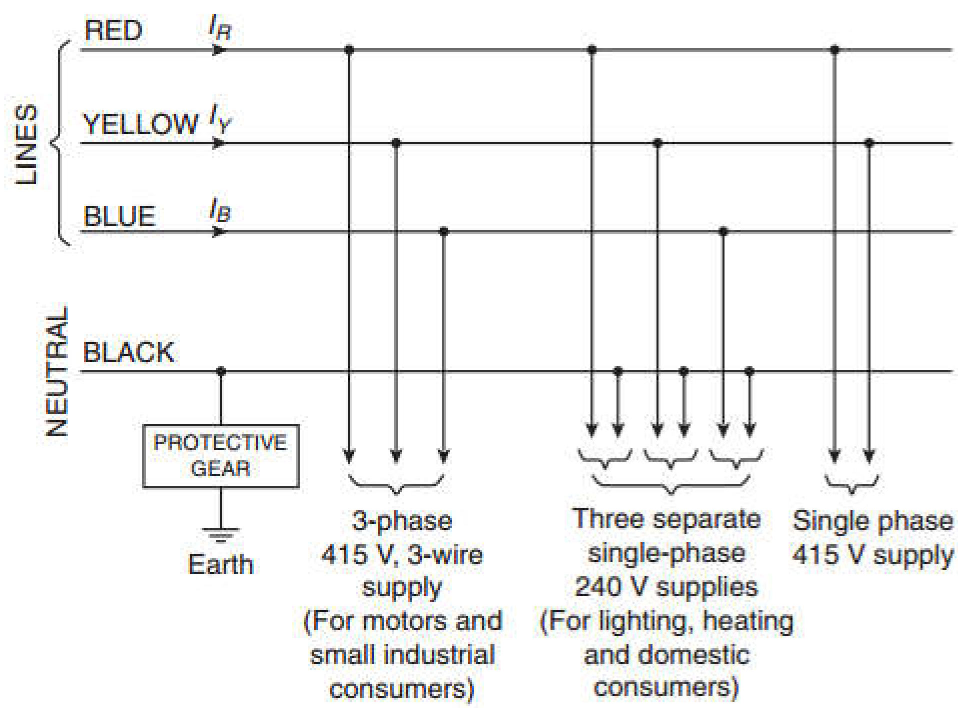


Figure 8.5

Example 8.1

Three loads, each of resistance 30, are connected in star to a 415V, three-phase supply. Determine (a) the system phase voltage, (b) the phase current and (c) the line current.

Solution:

A '415V, three-phase supply' means that 415V is the line voltage, V_L

$$For\ a\ star\ connection, V_L = \sqrt{3}V_p$$

$$Hence\ phase\ voltage, V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6V\ or\ \approx 240V$$

$$Phase\ current, I_P = \frac{V_p}{R_p} = \frac{240}{30} = 8A$$

$$For\ a\ star\ connection, I_p = I_L, hence\ the\ line\ current, I_L = 8A$$

Example 8.2

A star-connected load consists of three identical coils each of resistance 30 and inductance 127.3mH. If the line current is 5.08 A, calculate the line voltage if the supply frequency is 50Hz.

$$Inductive\ reactance\ X_L = 2\pi fL = 2\pi(50)(127.3 \times 10^{-3}) = 40\Omega$$

$$Impedance\ of\ each\ phase\ Z_p = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

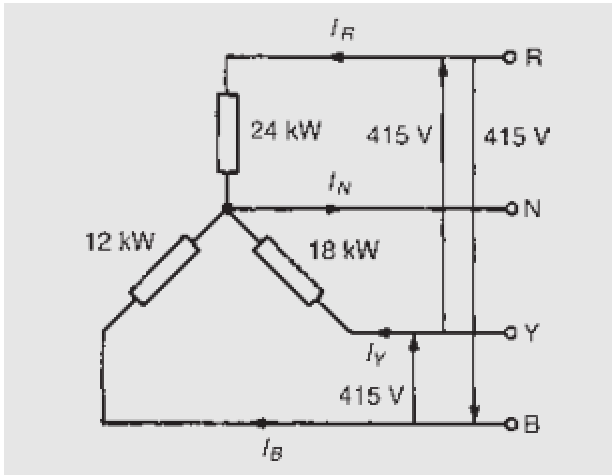
$$For\ a\ star\ connection\ I_L = I_p = \frac{V_p}{Z_p}$$

$$Hence\ phase\ voltage\ V_p = I_p Z_p = (5.08)(50) = 254V$$

$$Line\ voltage\quad V_L = \sqrt{3}V_p = \sqrt{3}(254) = 440V$$

Example 8.3

A 415V, three-phase, four wire, star-connected system supplies three resistive loads as shown in **Figure 8**. Determine (a) the current in each line and (b) the current in the neutral conductor.



$$For\ a\ star\ connected\ system, V_L = \sqrt{3}V_p$$

$$Hence \quad V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

$$Since\ current\ I = \frac{power\ P}{voltage\ V}\ for\ a\ resistive\ load$$

$$then\ I_R = \frac{P_R}{V_R} = \frac{24000}{240} = 100A$$

$$then\ I_Y = \frac{P_Y}{V_Y} = \frac{18000}{240} = 75A$$

$$then\ I_B = \frac{P_B}{V_B} = \frac{12000}{240} = 50A$$

The three line currents are shown in the phasor diagram of Figure 8.6. Since each load is resistive the currents are in phase with the phase voltages and are hence mutually displaced by 120^0. The current in the neutral conductor is given by:

$$I_N = I_R + I_Y + I_B\ phasorially$$

Figure 8.7 shows the three line currents added phasorially. O_a represents I_R in magnitude and direction. From the nose of O_a, *ab* is drawn representing I_Y in magnitude and direction. From the nose of *ab*, *bc* is drawn representing IB in magnitude and direction. O_c represents the resultant, IN

By measurement, I_N =43A

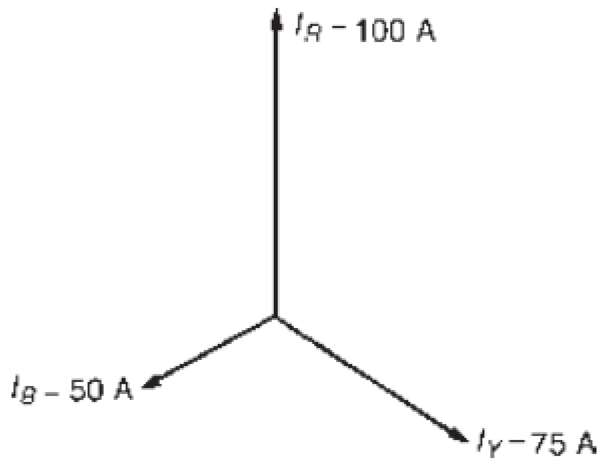


Figure 8.6

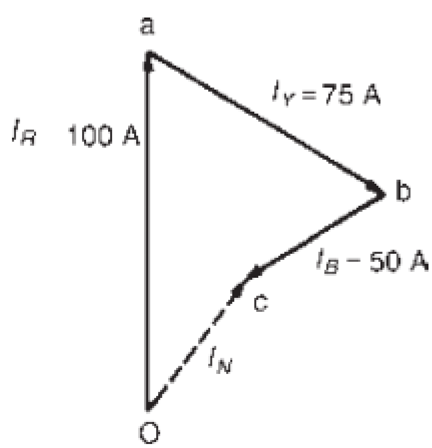


Figure 8.7

Alternatively, by calculation, considering I_R at 90^0, I_B at 210^0 and I_Y at 330^0:

$$Total\ horizontal\ component = 100cos90^0 + 75cos330^0 + 50cos210^0 = 21.65$$

$$Total\ vertical\ component = 100sin90^0 + 75sin330^0 + 50sin210^0 = 37.50$$

$$Hence\ magnitude\ of\ I_N = \sqrt{21.65^2 + 37.50^2} = 43.3A$$

8.3 Delta Connection

(i) A delta (or mesh) connected load is shown in **Figure 8.8** where the end of one load is connected to the start of the next load.

(ii) From **Figure 8.8**, it can be seen that the line voltages V_{RY}, V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_p$$

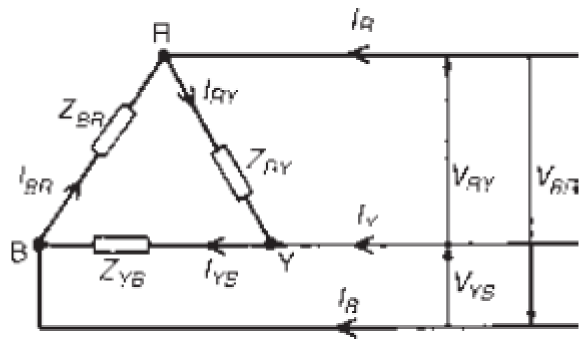


Figure 8.8

(iii) Using Kirchhoff's current law in Figure 8.8, $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$. From the phasor diagram shown in Figure 8.9, by trigonometry or by measurement, $I_R = \sqrt{3} I_{RY}$, i.e. for a delta connection:

$$I_L = \sqrt{3} I_p$$

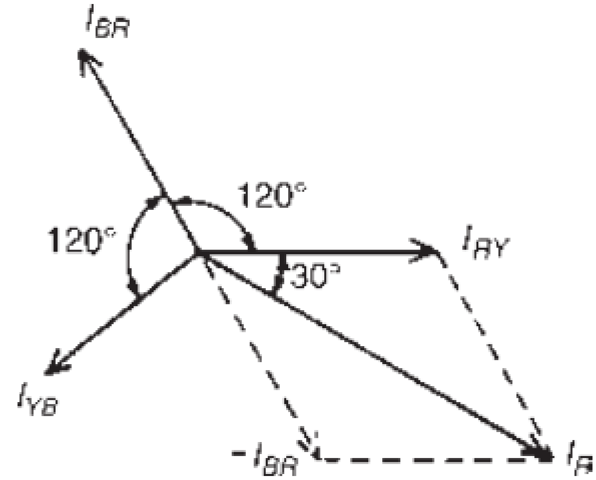


Figure 8.9

Example 8.4

Three identical coils each of resistance 30 and inductance 127.3 mH are connected in delta to a 440 V, 50 Hz, three-phase supply. Determine (a) the phase current, and (b) the line current

$$Inductive\ reactance\ X_L = 2\pi fL = 2\pi(50)(127.3 \times 10^{-3}) = 40\Omega$$

$$Impedance\ of\ each\ phase\ Z_p = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

$$Phase\ current,\ \ I_p = \frac{V_p}{V_p} = \frac{V_L}{Z_p} = \frac{400}{50} = 8.8A$$

$$For\ a\ delta\ connection, I_L = \sqrt{3}I_p = \sqrt{3}(8.8) = 15.24A$$

Thus when the load is connected in delta, three times the line current is taken from the supply than is taken if connected in star

Example 8.5

Three identical capacitors are connected in delta to a 415V, 50Hz, three-phase supply. If the line current is 15 A, determine the capacitance of each of the capacitors

$$For\ a\ delta\ connection\ I_L = \sqrt{3}I_p$$

$$Hence\ phase\ current\ \ I_p = \frac{I_L}{\sqrt{3}} = \frac{15}{\sqrt{3}} = 8.66A$$

$$Capacitive\ reactance\ per\ phase, X_C = \frac{V_p}{I_p} = \frac{V_L}{I_p} = \frac{415}{8.66} = 47.92\Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(47.92)}F = 66.43\mu F$$

8.4 Power in three phase systems

The power dissipated in a three-phase load is given by the sum of the power dissipated in each phase. If a load is balanced then the total power *P* is given by:

$$P = 3 \times power\ consumed\ by\ one\ phase$$

The power consumed in one phase:

$$P = I_p^2 R_p\ or\ V_p I_p cos\phi (where\ \phi\ is\ the\ phase\ angle\ between\ V_p\ and\ I_p)$$

$$For\ a\ star\ connection, V_p = \frac{V_L}{\sqrt{3}}\ and\ I_P\ hence$$

$$P = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L cos\phi = \sqrt{3} V_L I_L cos\phi$$

$$For\ a\ delta\ connection, V_p = V_L\ and\ I_p = \frac{I_L}{\sqrt{3}}\ hence$$

$$P = 3V_L\left(\frac{I_L}{\sqrt{3}}\right) I_L cos\phi = \sqrt{3} V_L I_L cos\phi$$

Hence for either a star or a delta balanced connection the total power *P* is given by:

$$\sqrt{3} V_L I_L cos\phi\ \ or\ P = 3I_p^2 R_p watts$$

$$Total\ volt - amperes, S = \sqrt{3} V_L I_L\ volt - amperes$$

Example 8.6

Three 12Ω resistors are connected in star to a 415V, three-phase supply. Determine the total power dissipated by the resistors.

$$Power\ dissipated, P = \sqrt{3}V_L I_L cos\phi\ or\ P = 3I_p^2 R_p$$

$$Line\ voltage, V_L = 415V\ and$$

$$phase\ voltage\ V_p = \frac{415}{\sqrt{3}} - 240V(since\ resistors\ are\ star - connected$$

$$Phase\ current, I_p = \frac{V_p}{Z_p} = \frac{V_p}{R_p} = \frac{240}{12} = 20A$$

$$For\ star\ connection\ I_L = I_P = 20A$$

$$For\ \ a\ purely\ resistive\ load, the\ power\ factor = cos\ \phi\ = 1$$

$$Hence\ power\ P = \sqrt{3}V_L I_L cos\phi = \sqrt{3}(415)(20)(1) = 14.4kW$$

$$or\ power\ P = 3I_p^2 R_p = 3 \times 20^2 \times 12 = 14.4kW$$

Example 8.7

The input power to a three-phase a.c. motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A, respectively, determine the power factor of the system

$$Power, P = 5000W; line\ voltage\ V_L = 500V; line\ current, I_L = 8.6A$$

$$Power, P = \sqrt{3}V_L I_L cos\phi$$

$$Hence\ power\ factor = cos\phi = \frac{P}{\sqrt{3}V_L I_L} = \frac{5000}{\sqrt{3} \times (400)(8.6)} = 0.839$$

Example 8.8

Three identical coils, each of resistance 10 and inductance 42mH are connected (a) in star and (b) in delta to a 415V, 50Hz, three-phase supply. Determine the total power dissipated in each case.

Star connection:

$$Inductive\ reactance\ X_L = 2\pi fL = 2\pi \times 42 \times 10^{-3} = 13.19\Omega$$

$$Phase\ impedance\ Z_p = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 13.19^2} = 16.55\Omega$$

$$Line\ voltage\ V_L = 415\ and$$

$$phase\ voltage, V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

$$Phase\ current, I_p = \frac{V_p}{Z_p} = \frac{240}{16.55} = 14.50A$$

$$Line\ current, I_L = I_p = 14.50A$$

$$Power\ factor = cos\phi = \frac{R_p}{Z_p} = \frac{10}{16.55} = 0.6042\ lagging$$

$$Power\ dissipated, P = \sqrt{3}V_L I_L cos\phi = \sqrt{3} \times 415 \times 14.50 \times 0.6042 = 6.3kW$$

$$Or\ \ P = 3I_p^2 R_p = 3 \times 14.50^2 \times 10 = 6.3kW$$

Delta Connection:

$$V_L = V_p = 415V, Z_p = 16.55\Omega, cos\phi = 0.6042\ lagging(from\ above)$$

$$Phase\ current, I_P = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08A$$

$$Line\ current, I_L = \sqrt{3}I_p = \sqrt{3} \times 25.08 = 43.44A$$

$$Power\ Dissipated, P = \sqrt{3}V_L I_L cos\phi = \sqrt{3} \times 415 \times 43.44 \times 0.6042 = 18.87kW$$

$$Or\ , P = 3I_p^2 R_p = 2 \times 25.08^2 \times 10 = 18.87kW$$

Hence loads connected in delta dissipate three times the power than when connected in star, and also take a line current three times greater.

8.5 Measurement of power in three phase systems

Power in three-phase loads may be measured by the following methods:

(i) **One-wattmeter method for a balanced load**

Total power = 3× wattmeter reading

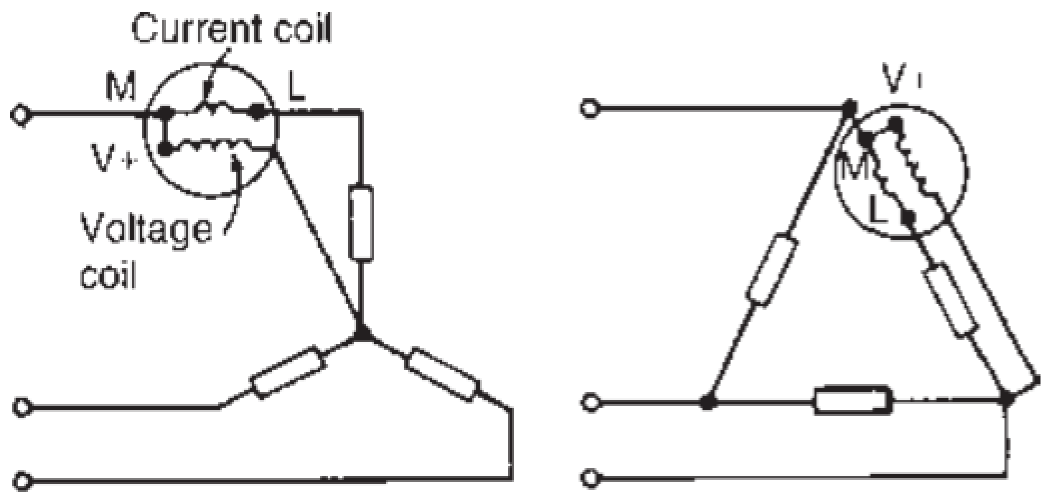


Figure 8.10

(ii) **Two-wattmeter method for balanced or unbalanced loads**

For a star-connected load

Total power=sum of wattmeter readings $=P_1+P_2$

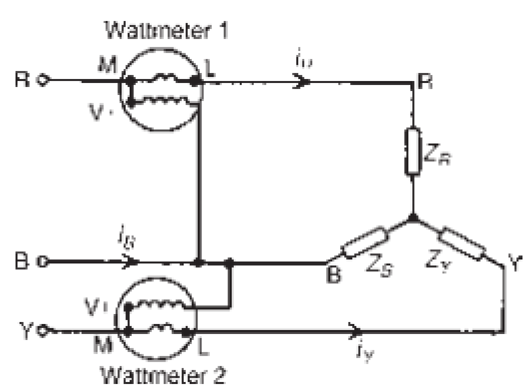


Figure 8.11

The power factor may be determined from:

$$tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

(iii) **Three-wattmeter method for a three-phase, four-wire system for balanced and unbalanced loads**

Total power = $P_1 + P_2 + P_3$

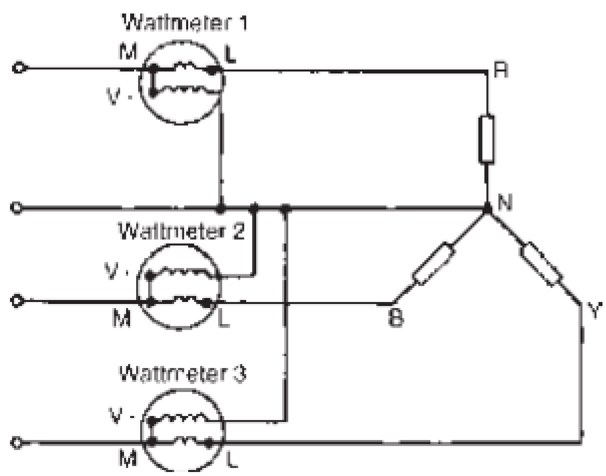


Figure 8.12

Example 8.9

Two wattmeters are connected to measure the input power to a balanced three-phase load by the two-wattmeter method. If the instrument readings are 8kW and 4kW, determine (a) the total power input and (b) the load power factor:

Solution:

$$a. Total\ Power, P = P_1 + P_2 = 8 + 4 = 12kW$$

$$b. tan\phi == \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) = \sqrt{3}\left(\frac{8-4}{8+4}\right) = \sqrt{3}\left(\frac{4}{12}\right) = \sqrt{3}\frac{1}{3} = \frac{1}{\sqrt{3}}$$

$$Hence\ \phi = tan^{-1}\frac{1}{\sqrt{3}} = 30^0$$

$$Power\ factor = cos\phi = cos30^0 = 0.866$$

8.6 Comparison of star and delta connected connections

(i) Loads connected in delta dissipate three times more power than when connected in star to the same supply.

(ii) For the same power, the phase currents must be the same for both delta and star connections (since, $P = 3I_p^2R_p$), hence the line current in the delta-connected system is greater than the line current in the corresponding star-connected system. To achieve the same phase current in a star-connected system as in a delta-connected system, the line voltage in the star system is √3 times the line voltage in the delta system. Thus for a given power transfer, a delta system is associated with larger line currents (and thus larger conductor cross-sectional area) and a star system is associated with a larger line voltage (and thus greater insulation).

8.7 Advantages of three phase systems

In addition to advantages mentioned in previous lectures other advantages of three-phase systems over single-phase supplies include:

(i) For a given amount of power transmitted through a system, the three-phase system requires conductors with a smaller cross-sectional area. This means a saving of copper (or aluminium) and thus the original installation costs are less.

(ii) Two voltages are available.

(iii) Three-phase motors are very robust, relatively cheap, generally smaller, have self-starting properties, provide a steadier output and require little maintenance compared with single-phase motors.