LECTURE 7-Single Phase AC Circuits

7.1 Purely resistive a.c circuit

In a purely resistive a.c. circuit, the current I_R and applied voltage V_R are in phase.

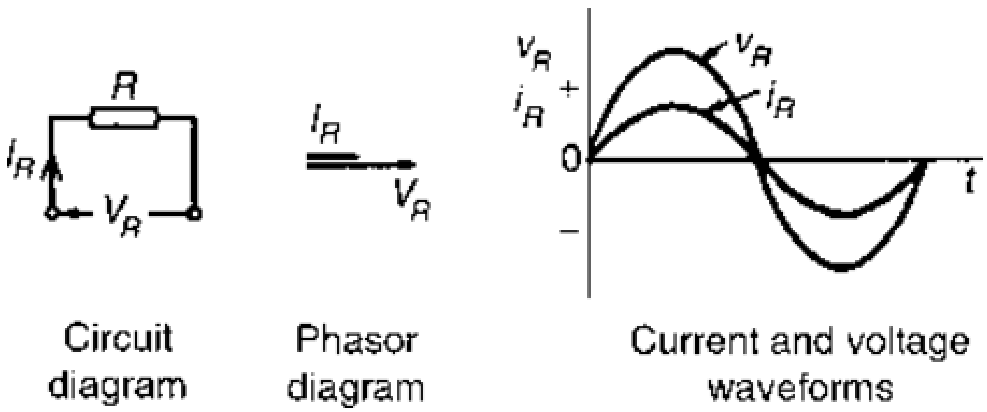

Figure 7.1

7.2 Purely inductive a.c circuit

In a purely inductive a.c. circuit, the current I_L lags the applied voltage V_L by 90◦ (i.e. $\pi/2$ rads).

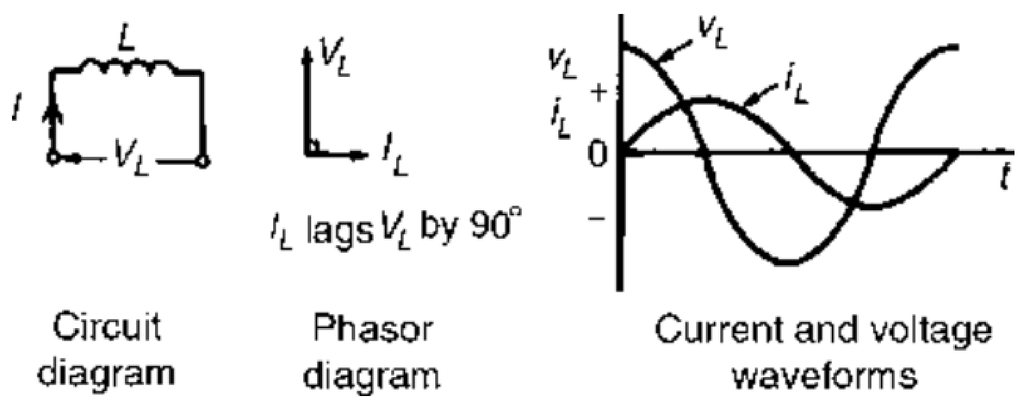

Figure 7.2

In a purely inductive circuit the opposition to the flow of alternating current is called the **inductive reactance, X_L**

$$X_L = \frac{V_L}{I_L} = 2\pi f L \Omega$$

where f is the supply frequency, in hertz, and L is the inductance, in henrys. X_L is proportional to f

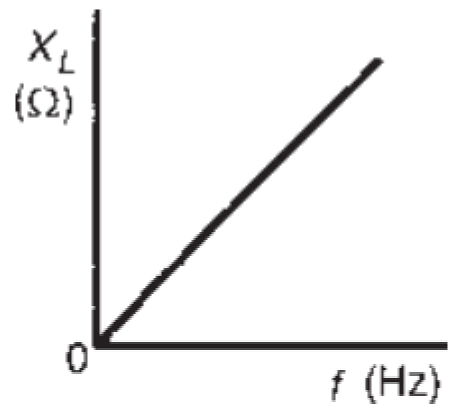

Figure 7.3

Example 7.1

(a) Calculate the reactance of a coil of inductance 0.32 H when it is connected to a 50 Hz supply.
(b) A coil has a reactance of 124 in a circuit with a supply of frequency 5 kHz. Determine the inductance of the coil.

$$a. Inductive\ reactance, X_L = 2\pi f L = 2\pi(50)(0.32) = 100.5\Omega$$

$$b. since\ X_L = 2\pi f L, inductance$$

$$L = \frac{X_L}{2\pi f} = \frac{124}{2\pi(5000)} H$$

7.3 Purely capacitive a.c circuit

In a purely capacitive a.c. circuit, the current I_C **leads** the applied voltage V_C by 90^0 (i.e. $\pi/2$ rads).

$$X_C = \frac{V_C}{I_C} = \frac{1}{2\pi f C}\Omega$$

where C is the capacitance in farads. X_C varies with frequency f as shown in figure 7.5

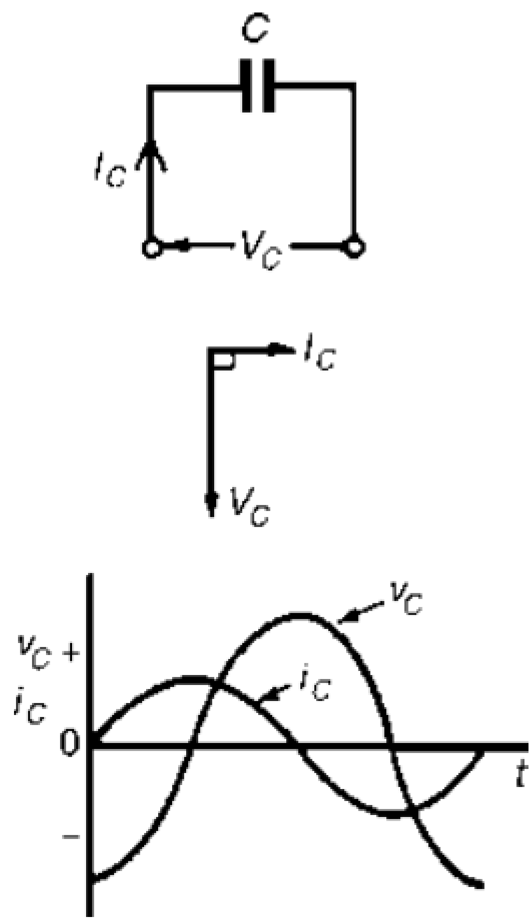

Figure 7.4

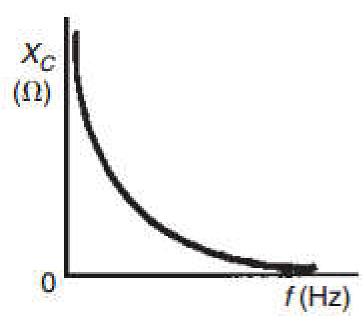

Figure 7.5

Example 7.2

A capacitor has a reactance of 40Ω when operated on a 50 Hz supply. Determine the value of its capacitance.

$$Since\ X_C = \frac{1}{2\pi f C}, capacitance\ C = \frac{1}{2\pi f X_c}$$

$$= \frac{1}{2\pi(50)(40)} F = \frac{10^6}{2\pi(50)(40)} \mu F = 79.58 \mu F$$

Example 7.3

Calculate the current taken by a 23μF capacitor when connected to a 240 V, 50 Hz supply.

$$Current\ I = \frac{V}{X_C} = \frac{V}{\left(\frac{1}{2\pi f C}\right)} = 2\pi f C V = 2\pi(50)(23 \times 10^{-6})(240) = 1.73A$$

CIVIL

The relationship between voltage and current for the inductive and capacitive circuits can be summarized using the word 'CIVIL', which represents the following: **in a capacitor (C) the current (I) is ahead of the voltage (V), and the voltage (V) is ahead of the current (I) for the inductor (L).**

7.4 R-L series ac. Circuit

In an a.c. circuit containing inductance *L* and resistance *R*, the applied voltage *V* is the phasor sum of V_R and V_L (See figure below)), and thus the current *I* lags the applied voltage *V* by an angle lying between 0^0 and 90^0 (depending on the values of V_R and V_L), shown as angle φ. In any a.c. series circuit the current is common to each component and is thus taken as the reference phasor.

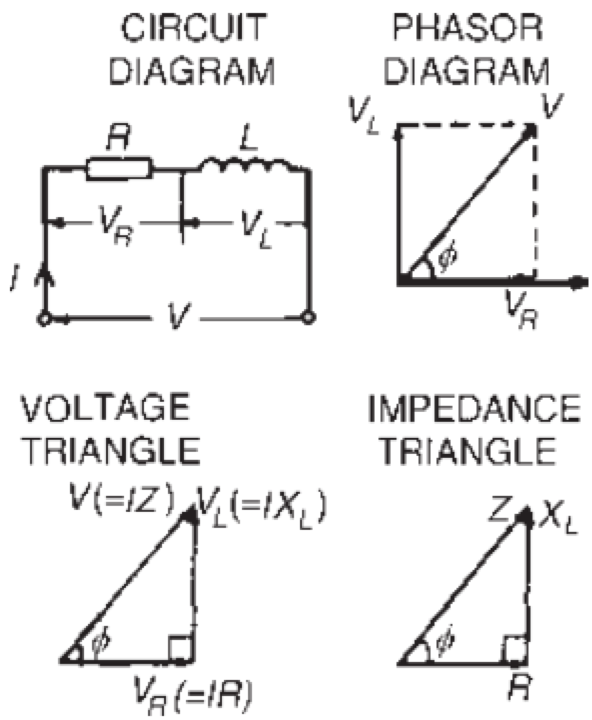

Figure 7.6

From the phasor diagram of Figure above, the 'voltage triangle' is derived.

For the *R–L* circuit:

$$V = \sqrt{V_R^2 + V_L^2}\ (by\ Pythagoras'theorem)$$

$$and\ tan\phi = \frac{V_L}{V_R}(by\ trigonometric\ ratios)$$

$$In\ an\ a.c\ circui, the\ ratio\ \frac{applied\ voltage\ V}{current\ I}\ is\ called\ the\ impedance\ Z, i.e$$

$$Z = \frac{V}{I}\Omega$$

If each side of the voltage triangle in Figure above is divided by current *I* then the **'impedance triangle'** is derived:

$$For\ the\ R - L\ circuit{:}\ Z = \sqrt{R^2 + X_L^2}$$

$$tan\phi = \frac{X_L}{R}, sin\phi = \frac{X_L}{Z} \text{ and } cos\phi = \frac{R}{Z}$$

Example 7.4

In a series *R–L* circuit the p.d. across the resistance *R* is 12 V and the p.d. across the inductance *L* is 5 V. Find the supply voltage and the phase angle between current and voltage.

$$Supply\ Voltage\ V = \sqrt{12^2 + 5^2} = 13V$$

(Note that in a.c. circuits, the supply voltage is **not** the arithmetic sum of the p.d.s (potential differences) across components. It is, in fact, the **phasor sum**.)

$$tan\phi = \frac{V_L}{V_R} = \frac{5}{12}, from\ which\ \phi = tan^{-1}\left(\frac{5}{12}\right) = 22.62^0\ lagging$$

('Lagging 'infers that the current is 'behind' the voltage, since phasors revolve anticlockwise.)

7.5 R-C series ac. Circuit

In an a.c. series circuit containing capacitance *C* and resistance *R*, the applied voltage *V* is the phasor sum of V_R and V_C (see Figure below) and thus the current *I* leads the applied voltage *V* by an angle lying between 0^0 and 90^0(depending on the values of V_R and V_C), shown as angle *α*. From the phasor diagram of Figure below, the **'voltage triangle'** is derived. For the *R–C* circuit:

$$V = \sqrt{V_R^2 + V_C^2}\ (by\ Pythagoras'theorem)$$

$$and\ tan\phi = \frac{V_C}{V_R}(by\ trigonometric\ ratios)$$

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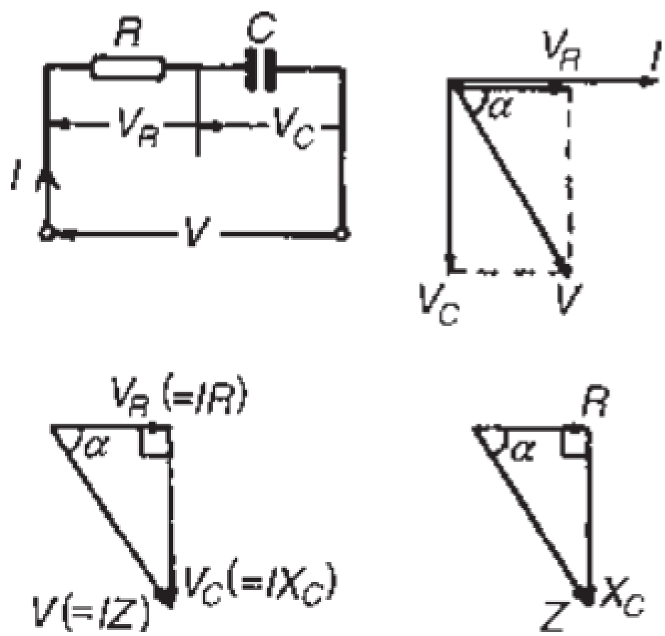


Figure 7.7

If each side of the voltage triangle in Figure above is divided by current *I* then the **'impedance triangle'** is derived:

$$For\ the\ R - L\ circuit: Z = \sqrt{R^2 + X_C^2}$$

$$tan\phi = \frac{X_C}{R}, sin\phi = \frac{X_C}{Z}\ and\ cos\phi = \frac{R}{Z}$$

Example 7.5

A resistor of 25 is connected in series with a capacitor of 45**µ**F. Calculate (a) the impedance and (b) the current taken from a 240 V, 50 Hz supply. Find also the phase angle between the supply voltage and the current.

R= **25;** C = **45**μ**F** = **45×10-6F;** V =**240 V;** f =**50 Hz**

$$Capacitance\ reactance, X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi(50)(45 \times 10^{-6})} = 70.74\Omega$$

$$a. Impedance\ Z = \sqrt{R^2 + X^2} = \sqrt{25^2 + 70.74^2} = 75.03\Omega$$

$$b. Current\ I = \frac{V}{Z} = \frac{240}{75.03} = 3.20A$$

Phase angle between the supply voltage and current,

$$\alpha = tan^{-1}\left(\frac{X_C}{R}\right)$$

$$hence\ \alpha = tan^{-1}\left(\frac{70.74}{25}\right) = 70.54^0\ leading$$

('Leading' infers that the current is 'ahead' of the voltage, since phasors revolve anticlockwise.)

7.6 R-L-C series a.c circuit

In an a.c. series circuit containing resistance R, inductance L and capacitance C, the applied voltage V is the phasor sum of V_R, V_L and V_C (see Figure below). V_L and V_C are anti-phase, i.e. displaced by 180^0, and there are three phasor diagrams possible – each depending on the relative values of V_L and V_C.

When $X_L > X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$and\ tan\phi = \frac{(X_L - X_C)}{R}$$

When $X_L > X_C$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$and\ tan\phi = \frac{(X_C - X_L)}{R}$$

When $X_L = X_C$ (Figure 7.8(d)), the applied voltage V and the current I are in phase. This effect is called series resonance

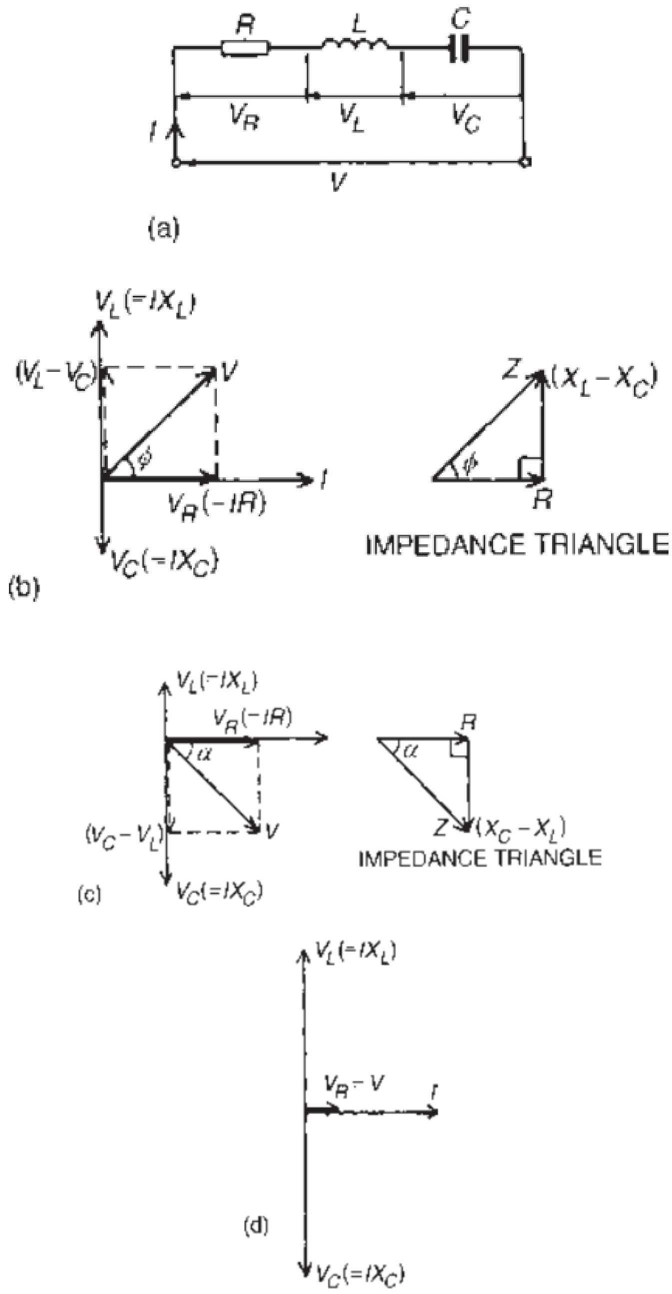
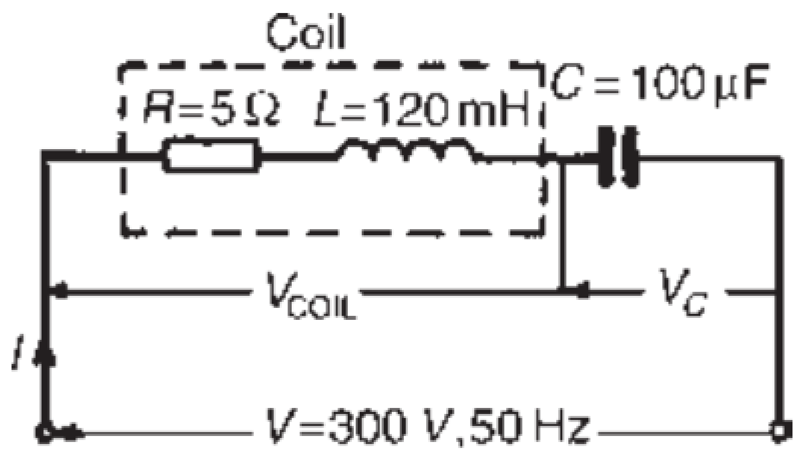


Figure 7.8

Example 7.6

A coil of resistance 5 and inductance 120 mH in series with a 100**μ**F capacitor is connected to a 300 V, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor



$$X_L = 2\pi f L = 2\pi(50)(120 \times 10^{-3}) = 37.70\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(120 \times 10^{-6})} = 31.83\Omega$$

$$X_L - X_C = 37.70 - 31.83 = 5.87\Omega$$

$$Impedance\ Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{5^2 + (5.87)^2} = 7.71\Omega$$

$$a. Current\ I = \frac{V}{Z} = \frac{300}{7.71} = 38.91A$$

$$b. Phase\ angle\ \phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right) = tan^{-1}\frac{5.87}{5} = 49.58^0$$

$$c. Impdeance\ pf\ coil, Z_{COIL}$$

$$= \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 37.70^2} = 38.03\Omega$$

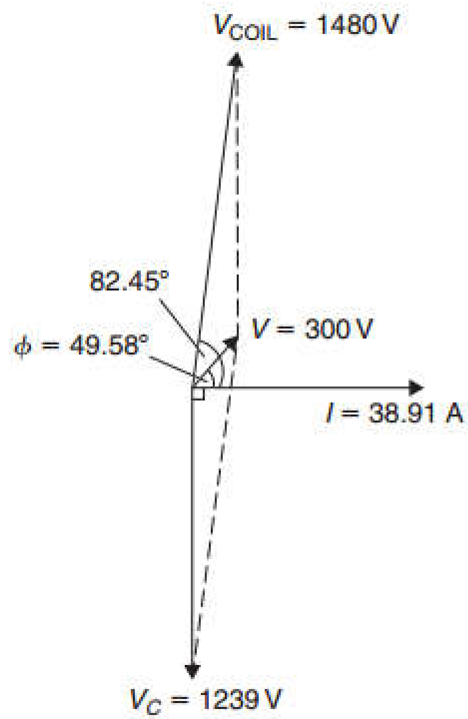
$$Voltage\ across\ coil\ V_{COIL} = IZ_{COIL}$$

$$= (38.91)(38.03) = 1480V$$

$$Phase\ angle\ of\ coil = tan^{-1}\frac{X_L}{R} = tan^{-1}\left(\frac{37.70}{5}\right)$$

$$Voltage\ across\ capacitor = V_C = IX_C = (38.91)(31.83) = 1239V$$

The phasor diagram is shown in Figure below. The supply voltage V is the phasor sum of V_{COIL} and V_C



7.7 Series resonance

As mentioned earlier for an $R–L–C$ series circuit, when $X_L = X_C$, the applied voltage V and the current I are in phase. This effect is called series resonance. At resonance:

$$a. V_L = V_C$$

$$b. Z = R(i.e\ the\ minimum\ circuit\ impedance\ possible\ in\ an\ L - C - R\ circuit).$$

$$c. I = \frac{V}{R}(i.e\ the\ maximum\ current\ possible\ in\ an\ L - C - R\ circuit).$$

$$d. Since\ X_L = X_C, then\ 2\pi f_r L = \frac{1}{2\pi f_r C}, from\ which, f_r^2 = \frac{1}{(2\pi)^2 LC}\ and$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} Hz, where\ f_r\ is\ the\ resonant\ frequency$$

$$e. The\ series\ resonant\ circuit\ is\ often\ described\ as\ an\ acceptor\ circuit\ since\ it$$

$$has\ its\ minimum\ impedance, and\ thus\ maximum\ current, at\ the\ resonant\ frequency$$

Example 7.7

A coil having a resistance of 10 and an inductance of 125 mH is connected in series with a 60μF capacitor across a 120 V supply. At what frequency does resonance occur? Find the current flowing at the resonant frequency.

$$Resonant\ frequency, f_r = \frac{1}{2\pi\sqrt{LC}} Hz = \frac{1}{2\pi\sqrt{\left(\frac{125}{10^3}\right)\left(\frac{60}{10^6}\right)}} Hz$$

$$= \frac{1}{2\pi\sqrt{\left(\frac{125 \times 6}{10^8}\right)}} Hz = 58.12 Hz$$

$$At\ resonance, X_L = X_C\ and\ impedance\ Z = R$$

$$Hence\ current, I = \frac{V}{R} = \frac{120}{10} = 12A$$

Exercise 7.1

The current at resonance in a series *L–C–R* circuit is 100**μ**A. If the applied voltage is 2 mV at a frequency of 200 kHz, and the circuit inductance is 50**μ**H, find (a) the circuit resistance, and (b) the circuit capacitance.

7.8 Q-Factor

At resonance, if R is small compared with X_L and X_C, it is possible for V_L and V_C to have voltages many times greater than the supply voltage.

$$Voltage\ magnification\ at\ resonance = \frac{Voltage\ across\ L(or\ C)}{Supply\ voltage\ V}$$

This ratio is a measure of the quality of a circuit (as a resonator or tuning device) and is called the **Q-factor.**

$$Hence\ Q - Factor = \frac{V_L}{V} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

$$Also, Q - Factor = \frac{V_C}{V} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi f_r CR}$$

$$At\ resonance, f_r = \frac{1}{2\pi\sqrt{LC}}\ i.e\ 2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$Hence\ Q - factor = \frac{2\pi f_r L}{R} = \frac{1}{\sqrt{LC}}\left(\frac{L}{R}\right) = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Exercise 7.2

A coil of inductance 80 mH and negligible resistance is connected in series with a capacitance of 0.25μF and a resistor of resistance 12.5 across a 100 V, variable frequency supply. Determine (a) the resonant frequency and (b) the current at resonance. How many times greater than the supply voltage is the voltage across the reactances at resonance?

Exercise 7.3

A series circuit comprises a coil of resistance 2 and inductance 60 mH, and a 30**μ**F capacitor. Determine the Q-factor of the circuit at resonance.

7.9 Bandwidth and selectivity

The Figure below shows how current I varies with frequency in an *R–L–C* series circuit. At the resonant frequency f_r , current is a maximum value, shown as I_r . Also shown are the points A and B where the current is 0.707 of the maximum value at frequencies f_1 and f_2. The power delivered to the circuit is I^2R. At I =0.707I_r the power is $(0.707I_r)^2R = 0.5I_r^2R$ i.e. half the power that occurs at frequency f_r . The points corresponding to f_1 and f_2 are called the *half-power points*. The distance between these points, i.e. (f_2 - f_1), is called the *bandwidth*.

It may be shown that

$$Q = \frac{f_r}{f_2 - f_1} = or\ (f_2 - f_1) = \frac{f_r}{Q}$$

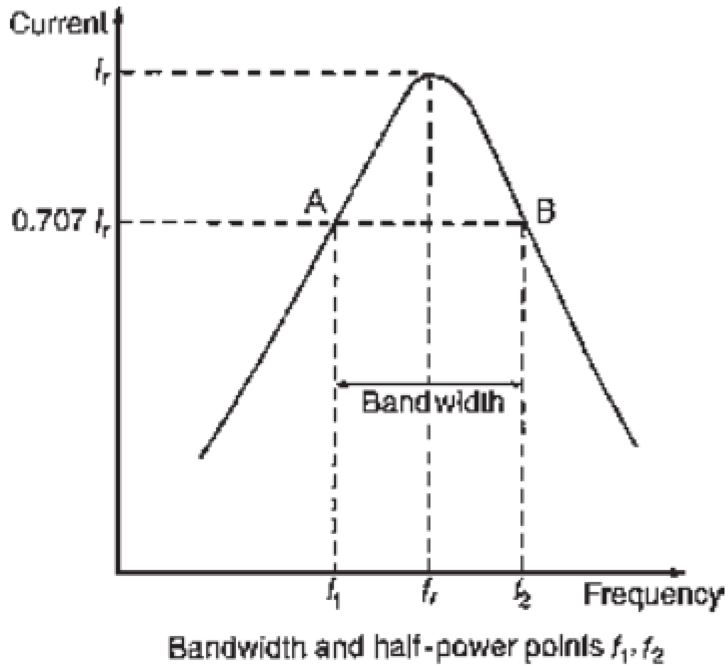


Figure 7.9

7.10 Selectivity

Selectivity *i*s the ability of a circuit to respond more readily to signals of a particular frequency to which it is tuned than to signals of other frequencies. The response becomes progressively weaker as the frequency departs from the resonant frequency. The higher the Q-factor, the narrower the bandwidth and the more selective is the circuit. Circuits having high Q-factors (say, in the order of 100 to 300) are therefore useful in communications engineering. A high Q-factor in a series power circuit has disadvantages in that it can lead to dangerously high voltages across the insulation and may result in electrical breakdown.

Example 7.8

A filter in the form of a series *L–R–C* circuit is designed to operate at a resonant frequency of 5 kHz. Included within the filter is a 20 mH inductance and 10Ω resistance. Determine the bandwidth of the filter.

Q-factor at resonance is given by

$$Q_r = \frac{w_r L}{R} = \frac{(2\pi 5000)(20 \times 10^{-3})}{10}$$

$$Since\ Q_r = \frac{f_r}{(f_2 - f_1)}$$

$$bandwidth, (f_2 - f_1) = \frac{f_r}{(Q_r)} = \frac{5000}{62.83} = 79.6Hz$$

7.11 Power in ac. Circuits

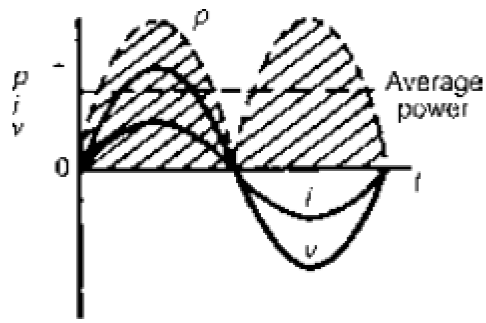
In Figures below the value of power at any instant is given by the product of the voltage and current at that instant, i.e. the instantaneous power, $p=vi$, as shown by the broken lines.

(a) For a purely resistive a.c. circuit, the average power dissipated, *P*, is given by:

$$P = VI = I^2R = \frac{V^2}{R}\ watts$$

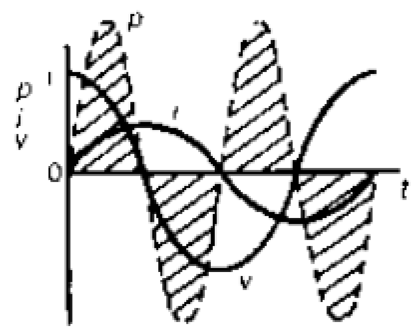
(b) For a purely inductive a.c. circuit, the average power is zero

(c) For a purely capacitive a.c. circuit, the average power is zero.



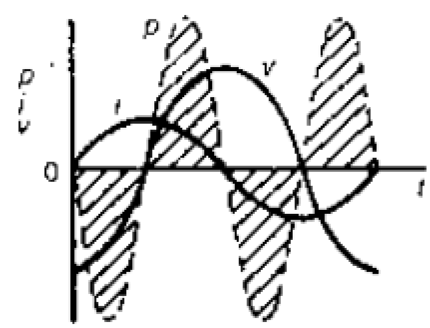
PURE RESISTANCE – AVERAGE POWER VI

(a)



PURE INDUCTANCE – AVERAGE POWER - 0

(b)



PURE CAPACITANCE – AVERAGE POWER = 0

(c)

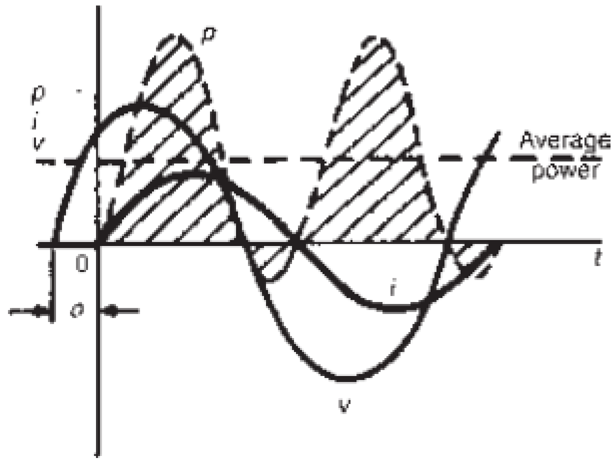


Figure 7.10

Example 7.9

A series circuit of resistance 60 and inductance 75 mH is connected to a 110 V, 60 Hz supply. Calculate the power dissipated.

Inductive reactance,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{60^2 + 28.27^2} = 66.33\Omega$$

$$Current, I = \frac{V}{Z} = \frac{100}{66.33} = 1.658A$$

To calculate power dissipation in an a.c. circuit two formulae may be used:

$$P = I^2R = (1.658^2)(60) = 165W$$

$$P = VIcos\phi \; where \; cos\phi = \frac{R}{Z}$$

$$= \frac{60}{66.33} = 0.9046$$

$$Hence \; P = (110)(1.658)(0.9046) = 165W$$

7.12 Power Triangle and Power Factor

The Figure below shows a phasor diagram in which the current I lags the applied voltage V by angle φ. The horizontal component of V is $V\ \cos\varphi$ and the vertical component of V is $V\ \sin\varphi$. If each of the voltage phasors is multiplied by I, .The Figure 7.11(b) is obtained and is known as the **'power triangle'**.

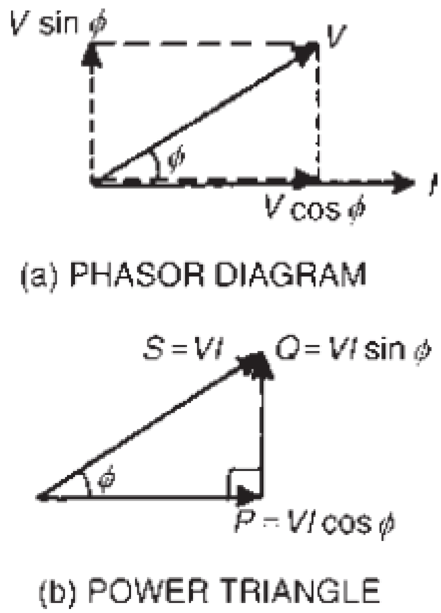


Figure 7.11

$$Apparent\ Power, S = VI\ voltamperes\ (VA)$$

$$True\ or\ active\ power, P = VI\ cos\phi\ watts(W)$$

$$Reactive\ power, Q = VIsin\phi\ reactive\ voltamperes(var)$$

$$Power\ factor = \frac{true\ power, P}{apparent\ power, S}$$

For sinusoidal voltages and currents,

$$Power\ factor = \frac{P}{S} = \frac{VIcos\phi}{VI}, i.e$$

$$p.f = cos\phi = \frac{R}{Z}$$

The relationships stated above are also true when current I leads voltage V

Example 7.10

A pure inductance is connected to a 150 V, 50 Hz supply, and the apparent power of the circuit is 300 VA. Find the value of the inductance.

$$Apparent\ power, S = VI$$

$$Hence\ current\ I = \frac{S}{V} = \frac{300}{150} = 2A$$

$$Inductive\ reactance\ X_L = \frac{V}{I} = \frac{150}{2} = 75\Omega$$

$$Since\ X_L = 2\pi fL, inductance\ L = \frac{X_L}{2\pi f} = \frac{75}{2\pi(50)} = 0.239$$

A transformer has a rated output of 200 kVA at a power factor of 0.8. Determine the rated power output and the corresponding reactive power.

$$VI = 200kVA = 200 \times 10^3; p.f = 0.8 = cos\phi$$

$$Power\ output, P = VIcos\phi = (200 \times 10^3)(0.8) = 160kW$$

$$Reactive\ power, Q = VIsin\phi$$

$$If\ cos\phi = 0.8, then\phi = cos^{-1}0.8 = 36.87^0$$

$$Hence\ sin\phi = sin36.87^0 = 0.6$$

$$Hence\ reactive\ power, Q = (200 \times 10^3)(0.6) = 120kvar$$

7.13 Single Phase parallel ac circuits

In parallel circuits, such as those shown in Figures below, the voltage is common to each branch of the network and is thus taken as the reference phasor when drawing phasor diagrams

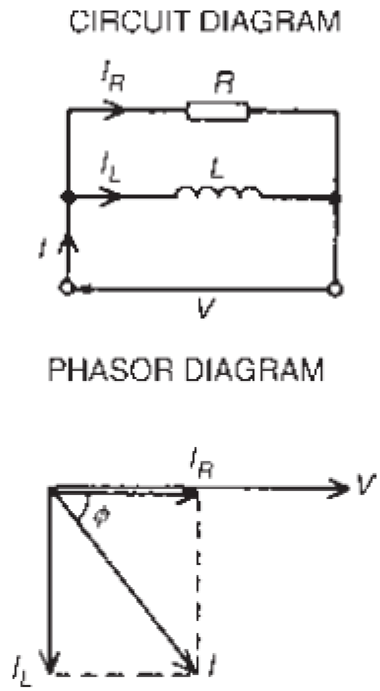


Figure 7.12

The formulae are the same as for series a.c. circuits.

7.14 R-L Parallel a.c circuit

In the two-branch parallel circuit containing resistance R and inductance L shown in Figure 7.12, the current flowing in the resistance, I_R, is in-phase with the supply voltage V and the current flowing in the inductance, I_L, lags the supply voltage by 90^0. The supply current I is the phasor sum of I_R and I_L and thus the current I lags the applied voltage V by an angle lying between 0^0 and 90^0 (depending on the values of I_R and I_L), shown as angle φ in the phasor diagram.

From the phasor diagram:

$$I = \sqrt{I_R^2 + I_L^2}$$

$$Where, I_R = \frac{V}{R} \; and \; I_L = \frac{V}{X_L}$$

$$tan\phi = \frac{I_L}{I_R}, sin\,\phi = \frac{I_L}{I} \; and$$

$$cos\phi = \frac{I_R}{I}(by\ trigonometric\ rations)$$

$$Circuit\ impedance, Z = \frac{V}{I}$$

Example 7.11

A 20Ω resistor is connected in parallel with an inductance of 2.387 mH across a 60 V, 1 kHz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, and (e) the power consumed.

$$a. Current\ flowing\ in\ the\ resistor\ I_R = \frac{V}{R} = \frac{60}{20} = 3A$$

$$Current\ flowing\ in\ the\ inductance$$

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi fL} = \frac{60}{2\pi(1000)(2.387 \times 10^{-3})} = 4A$$

$$b. From\ the\ phasor\ diagram, supply\ current,$$

$$= \sqrt{I_R^2 + I_L^2} = \sqrt{3^2 + 4^2} = 5A$$

$$c. Circuit\ phase\ angle, \phi = tan^{-1}\frac{I_L}{I_R} = tan^{-1}\left(\frac{4}{3}\right) = 53.13^0 lagging$$

$$d. Circuit\ impedance, Z = \frac{V}{I} = \frac{60}{5} = 12\Omega$$

$e. Power\ Consumed\ P = VIcos\phi = (60)(5)(cos53.13^0) = 180W$ or

$$Power\ Consumed, P = I_R^2R = (3^2)(20) = 180W$$

7.15 R-C Parallel a.c circuit

In the two-branch parallel circuit containing resistance *R* and capacitance *C* shown in Figure 7.2, I_R is in-phase with the supply voltage *V* and the current flowing in the capacitor, *IC* , leads *V* by 90^0. The supply current *I* is the phasor sum of I_R and I_C and thus the current *I* leads the applied voltage *V* by an angle lying between $0^0$and 90^0 (depending on the values of I_R and I_C), shown as angle α in the phasor diagram.

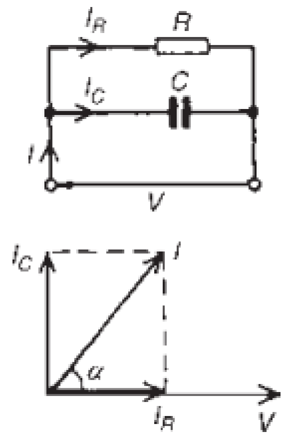


Figure 7.13

From the phasor diagram:

$$I = \sqrt{I_R^2 + I_L^2}$$

$$Where, I_R = \frac{V}{R} \; and \; I_C = \frac{V}{X_C}$$

$$tan\phi = \frac{I_C}{I_R}, sin\,\phi = \frac{I_C}{I} \; and$$

$$cos\phi = \frac{I_R}{I}(by\ trigonometric\ rations)$$

$$Circuit\ impedance, Z = \frac{V}{I}$$

Example 7.12

A 30μF capacitor is connected in parallel with an 80 resistor across a 240 V, 50 Hz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, (e) the power dissipated and (f) the apparent power.

$$a. Current\ flowing\ in\ the\ resistor\ I_R = \frac{V}{R} = \frac{240}{80} = 3A$$

$$Current\ flowing\ in\ the\ inductance$$

$$I_C = \frac{V}{X_C} = \frac{V}{\frac{1}{2\pi fC}} = 2\pi fC = 2\pi(50)(30 \times 10^{-6})(240) = 2.262A$$

$$b. From\ the\ phasor\ diagram, supply\ current,$$

$$= \sqrt{I_R^2 + I_C^2} = \sqrt{3^2 + 2.262^2} = 3.757A$$

$$c. Circuit\ phase\ angle, \phi = tan^{-1}\frac{I_C}{I_R} = tan^{-1}\left(\frac{2.262}{3}\right) = 37.02^0 leading$$

$$d. Circuit\ impedance, Z = \frac{V}{I} = \frac{240}{3.757} = 63.88\Omega$$

$$e. True\ or\ active\ power\ dissipated\ P = VIcos\phi = (240)(3.757)(cos37.02^0) = 720W$$

$$Alternatively, true\ power\ , P = I_R^2 R = (3^2)(80) = 720W$$

$$f. Apparent\ power, S = VI = (240)(3.757) = 901.7VA$$

7.16 L-C parallel a.c circuit

In the two-branch parallel circuit containing inductance *L* and capacitance *C* shown in Figure 7.14, I_L lags *V* by 90^0 and I_C leads *V* by 90^0 .Theoretically there are three phasor diagrams possible – each depending on the relative values of I_L and I_C:

(i) $I_L > I_C$ (giving a supply current, $I = I_L - I_C$ lagging *V* by 90^0)

(ii) $I_C > I_L$ (giving a supply current, $I = I_C - I_L$ leading *V* by 90^0)

(iii) $I_L = I_C$ (giving a supply current, I =0)

The latter condition is not possible in practice due to circuit resistance inevitably being present.

$$For\ the\ L - C\ parallel\ circuit, I_L = \frac{V}{X_L}, I_L = \frac{V}{X_L},$$

$$I = phasor\ difference\ between\ I_L\ and\ I_C, and\ Z = \frac{V}{I}$$

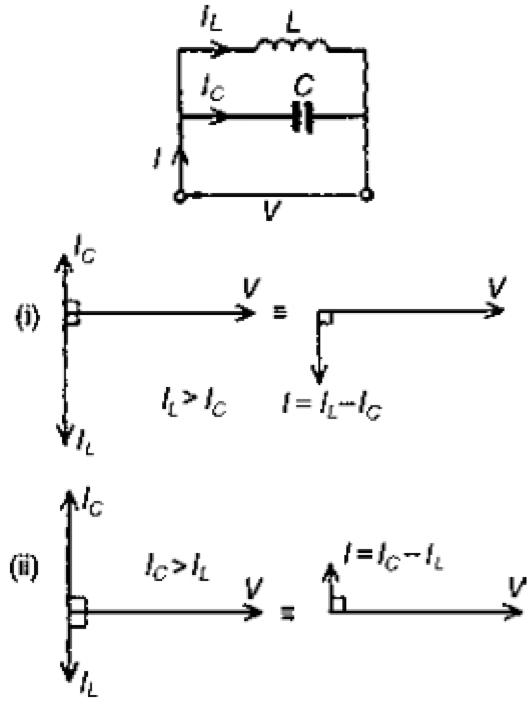


Figure 7.14

Example 7.13

A pure inductance of 120 mH is connected in parallel with a 25 μF capacitor and the network is connected to a 100 V, 50 Hz supply. Determine (a) the branch currents, (b) the supply current and its phase angle, (c) the circuit impedance and (d) the power consumed.

$$a. Inductive\ reactance, X_L = 2\pi f L = 2\pi(50)(120 \times 10^{-3}) = 37.70\Omega$$

$$Capacitive\ reactance, X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(25 \times 10^{-6})} = 127.3\Omega$$

$$Current\ flowing\ in\ inductance, I_L = \frac{V}{X_L} = \frac{100}{37.70} = 2.653A$$

$$Current\ flowing\ in\ inductance, I_L = \frac{V}{X_C} = \frac{100}{127.3} = 0.786A$$

$$b. I_L\ and\ I_C\ are\ anti-phase. Hence\ suply\ current,$$

$$I = I_L - I_C = 2.653 - 0.786 = 1.867A\ and\ the\ current\ lags\ the\ supply\ voltage\ by\ 90^0$$

7.17 LR-C parallel ac. Circuit

In the two-branch circuit containing capacitance *C* in parallel with inductance *L* and resistance *R* in series (such as a coil) shown in Figure 7.15(a), the phasor diagram for the *LR* branch alone is shown in Figure 7.15(b) and the phasor diagram for the *C* branch is shown alone in Figure 7.15(c) Rotating each and superimposing on one another gives the complete phasor diagram shown in Figure 7.15(d). The current I_{LR} of Figure 7.15(a) may be resolved into horizontal and vertical components. The horizontal component, shown as *op*, is $I_{LR}\cos\varphi1$ and the vertical component, shown as *pq*, is $I_{LR}\sin\varphi1$. There are three possible conditions for this circuit:

(i) $I_C > I_{LR}\sin\varphi1$ (giving a supply current *I* leading *V* by angle φ – as shown in Figure 7.15(e))

(ii) $I_{LR}\sin\varphi1 > I_C$ (giving *I* lagging *V* by angle φ –as shown in Figure 7.15(f))

(iii) $I_C = I_{LR}\sin\varphi1$ (this is called parallel resonance)

There are two methods of finding the phasor sum of currents *ILR* and *IC* in Figures 7.15(e) and (f). These are: (i) by a scaled phasor diagram, or (ii) by resolving each current into their *'in-phase'* (i.e. horizontal) and *'quadrature'* (i.e. vertical) *components.*

$$Impedance\ of\ LR\ branch\ , Z_{LR} = \sqrt{R^2 + X_L^2}$$

$$Current, I_{LR} = \frac{V}{Z_{LR}}\ \ and\ I_C = \frac{V}{X_C}$$

Supply current I =phasor sum of I_{LR} and I_C (by drawing)

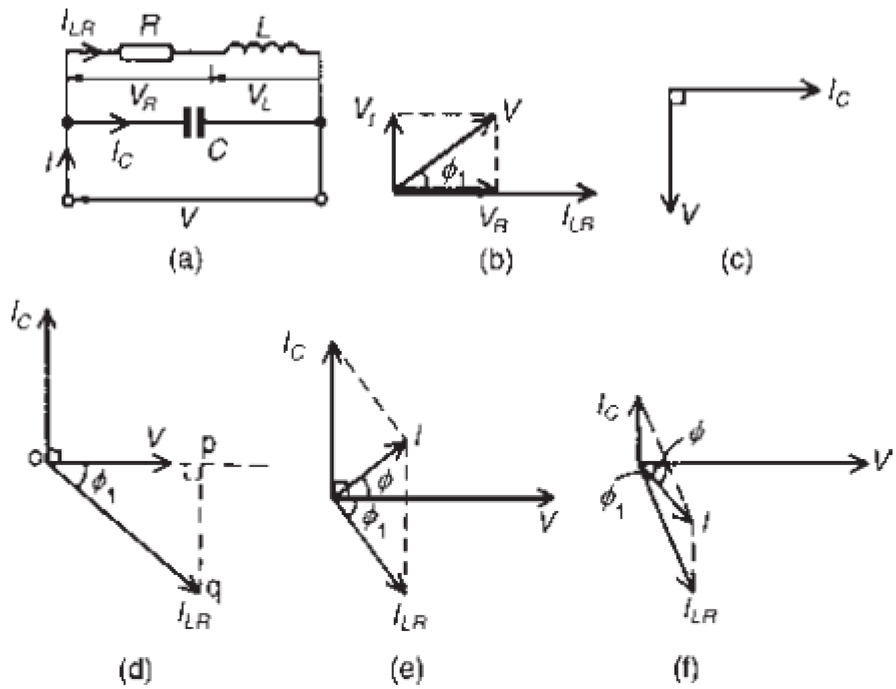
$$= \sqrt{(I_{LR}cos\phi_1)^2 + (I_{LR}sin\phi_1 \sim I_C)^2}$$

where ~ means 'the difference between'.

$$Circuit\ impedance\ Z = \frac{V}{I}$$

$$tan\phi_1 = \frac{V_L}{V_R} = \frac{X_L}{R}, sin\phi_1 = \frac{X_L}{Z_{LR}}\ and\ cos\phi_1 = \frac{R}{Z_{LR}}$$

$$tan\phi = \frac{I_{LR}sin\phi_1 \sim I_C}{I_{LR}cos\phi_1}\ and\ cos\phi = \frac{I_{LR}cos\phi_1}{I}$$



I_{LR}
R
L
V_R
V_L
I_C
C
I
V
(a)
V_1
V
ϕ_1
V_R
I_{LR}
(b)
I_C
V
(c)
I_C
o
V
p
ϕ_1
q
I_{LR}
(d)
I_C
I
ϕ
ϕ_1
V
I_{LR}
(e)
I_C
ϕ
V
ϕ_1
I
I_{LR}
(f)

Figure 7.15