LECTURE 3: Capacitors and Inductors

Capacitors and inductors do not dissipate but store energy, which can be retrieved later.

For this reason, capacitors and inductors are called storage elements.

3.1 Capacitors

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems. A capacitor can store energy, so capacitors are often found in power supplies. Capacitors are used for timing – for example with a 555 timer IC controlling the charging and discharging; for smoothing – for example in a power supply; for coupling – for example between stages of an audio system and a loudspeaker; for filtering – for example in the tone control of an audio system; for tuning – for example in a radio system; and for storing energy – for example in a camera flash circuit. Capacitors find uses in virtually every form of electronics circuit from analogue circuits, including amplifiers and power supplies, through to oscillators, integrators and many more.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

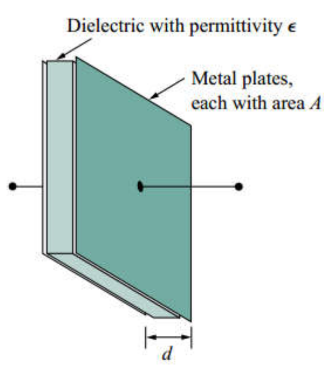


Figure 3.1

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

When a voltage source v is connected to the capacitor, as in Fig.3.2, the source deposits a positive charge q on one plate and a negative charge -q on the other. The capacitor is said to store the electric charge.

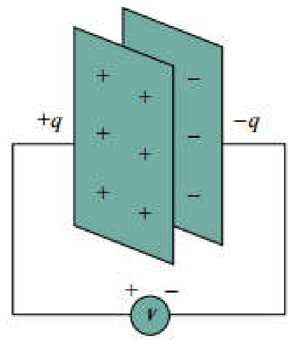


Figure 3.2

The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that,

$$q = Cv$$

Where C, the constant of proportionality, is known as the capacitance of the capacitor.

Thus**, Capacitance** is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Note from the equation above, 1 farad = 1 coulomb/volt.

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig. 6.1, the capacitance is given by

$$C = \frac{\epsilon A}{d}$$

Where A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric material between the plates.

In general, three factors determine the value of the capacitance:

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

Capacitors are commercially available in different values and types. Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range.

They are described by the dielectric material they are made of and by whether they are of fixed or variable type.

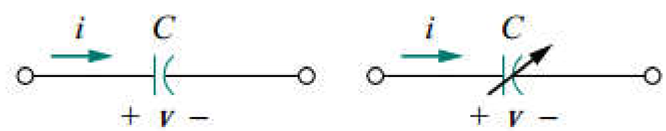


Figure 3.3: Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

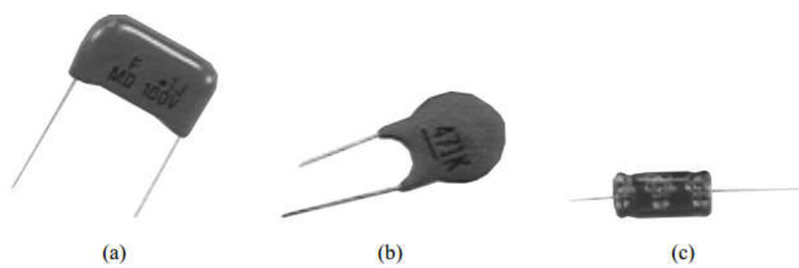


Figure 3.4: Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.

3.1.2: Energy Stored in a capacitor:

$$w = \frac{1}{2}Cv^2$$

$$w = \frac{q^2}{2C}$$

3.1.2: Properties of a capacitor

When the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus, a capacitor is an open circuit to DC.

The voltage on the capacitor must be continuous. The voltage on a capacitor cannot change abruptly.

The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

A real, nonideal capacitor has a parallel-model leakage resistance.

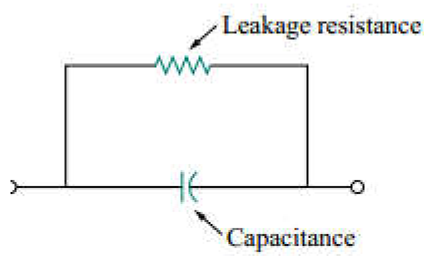


Figure 3.5: Circuit model of a nonideal capacitor.

Example 3.1

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

$$(a) Since\ q = Cv,$$

$$q = 3 \times 10^{-12} \times 20 = 60pC$$

$$(b) The\ energy\ stored\ is;$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600pJ$$

Exercise 3.1

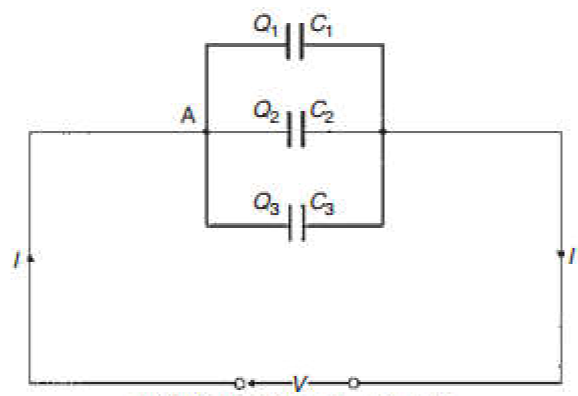
What is the voltage across a 3-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored?

Capacitors connected in parallel and series

(a) Capacitors connected in parallel

Figure 3.6 shows three capacitors, C_1, C_2 and C_3, connected in parallel with a supply voltage V applied across the arrangement.

When the charging current I reach point A it divides, some flowing into C_1, some flowing into C_2 and some into C_3. Hence the total charge Q_T (= I ×t) is divided between the three capacitors. The capacitors each store a charge and these are shown as Q_1, Q_2 and Q_3, respectively.



Total charge, $Q_T = Q_1 + Q_2 + Q_3$

Figure 3.6

Hence,

$$Q_T = Q_1 + Q_2 + Q_3$$

But $Q_T = CV$, $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$

Therefore, $CV = C_1V + C_2V + C_3V$, where C is the total equivalent circuit capacitance, i.e.

$$C_T = C_1 + C_2 + C_3$$

It follows that for n parallel-connected capacitors,

$$C_T = C_1 + C_2 + C_3 + \cdots C_n$$

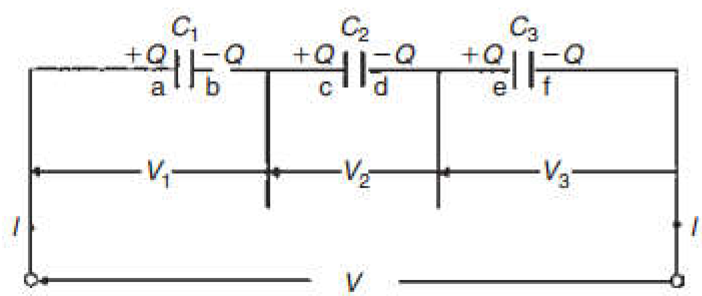
i.e. the equivalent capacitance of a group of parallel connected capacitors is the sum of the capacitances of the individual capacitors. (Note that this formula is similar to that used for resistors connected in series.)

(b) Capacitors connected in series

Figure 3.7 shows three capacitors, C_1, C_2 and C_3, connected in series across a supply voltage V. Let the potential difference (p.d) across the individual capacitors be V_1, V_2 and V_3, respectively, as shown.

Let the charge on plate 'a' of capacitor C_1 be +Q coulombs. This induces an equal but opposite

charge of −Q coulombs on plate 'b'. The conductor between plates 'b' and 'c' is electrically isolated from the rest of the circuit so that an equal but opposite charge of +Q coulombs must appear on plate 'c', which, in turn, induces an equal and opposite charge of –Q coulombs on plate 'd', and so on.



Charge on each capacitor = Q

Figure 3.7

Hence when capacitors are connected in series the charge on each is the same.

In a series circuit: $V = V_1 + V_2 + V_3$

$$Since\ V = \frac{Q}{C}\ then\ \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

where C is the total equivalent circuit capacitance,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

It follows that for n series-connected capacitors:

$$\frac{1}{C}\ then\ \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots+\frac{1}{C_n}$$

i.e. for series-connected capacitors, the reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances. (Note that this formula is similar to that used for resistors connected in parallel.)

For the special case of two capacitors in series:

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{C_1C_2}{C_2+C_1}\left(i.e\ \frac{product}{sum}\right)$$

Calculate the equivalent capacitance of two capacitors of 6μF and 4μF connected (a) in parallel and (b) in series

(a) In parallel, equivalent capacitance:

C =C1 +C2 =6μF+4μF=10μF

(b) In series, equivalent capacitance *C* is given by:

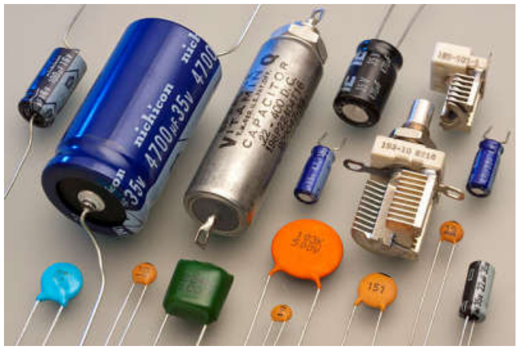
$$C=\frac{C_1C_2}{C_2+C_1}$$

This formula is used for the special case of two capacitors in series.

$$Thus\ C=\frac{6\times 4}{6+4}=\frac{24}{10}=2.4\mu$$

Exercise 3.2:

What capacitance must be connected in series with a 30μF capacitor for the equivalent capacitance to be 12μF?



3.8 Capacitors

3.2: Inductor

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors. Many common devices rely on magnetism. Familiar examples include computer disk drives, tape recorders, VCRs, transformers, motors, generators and so on. Practically all transformers and electric machinery uses magnetic material for shaping and directing the magnetic fields which act as a medium for transferring and connecting energy.

Any conductor of electric current has inductive properties and may be regarded as an inductor.

But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire

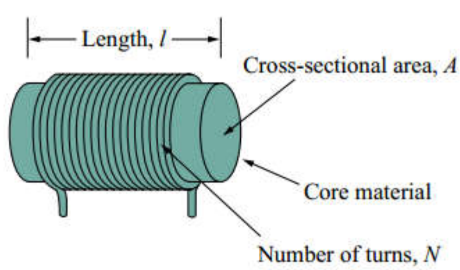


Figure 3.9: Typical form of an inductor.

An inductor consists of a coil of conducting wire.

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

$$v = L\frac{di}{dt}$$

Where L is the constant of proportionality called the inductance of the inductor.

The unit of inductance is the henry (H)

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H)

The inductance of an inductor depends on its physical dimension and construction.

For example, for the inductor (solenoid) shown in figure 3.9.

$$L = \frac{N^2\mu A}{\ell}$$

Where, L is inductance of coil in Henrys, N is the number of turns, ℓ is the length, A is the cross-sectional area of the coil, and μ is the permeability of the core. Note $\mu = \mu_r\mu_0$, μ_r is relative permeability of the core material (1 for air), μ_0 is permeability of free space.

We can see from the equation above that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil.

Like capacitors, commercially available inductors come in different values and types. Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems. Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms coil and choke are also used for inductors.

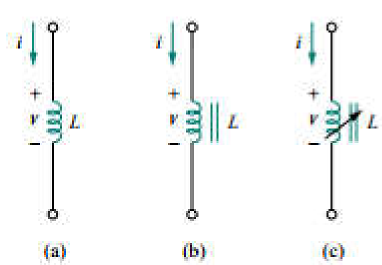


Figure 3.10: Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core

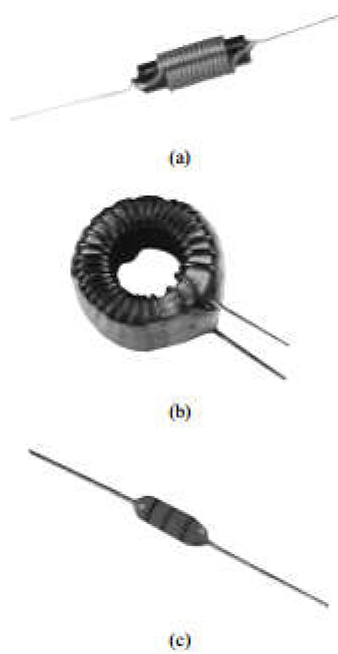


Figure 3.11: Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor

3.2.1 Energy stored in an inductor:

$$w = \frac{1}{2}Li^2$$

Where L is inductance, i is the current.

Properties of an inductor

The voltage across an inductor is zero when the current is constant thus an inductor acts like a short circuit to DC. An important property of the inductor is its opposition to the change in current flowing through it. The current through an inductor cannot change instantaneously.

Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved later. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

A practical, nonideal inductor has a significant resistive component. This is because the inductor is made of a conducting material such as copper, which has some resistance.

3.3: Applications

Capacitors and inductors possess the following three special properties that make them very useful in electric circuits:

1. The capacity to store energy makes them useful as temporary voltage or current sources. Thus, they can be used for generating a large amount of current or voltage for a short period.

2. Capacitors oppose any abrupt change in voltage; while inductors oppose any abrupt change in current. This property makes inductors useful for spark or arc suppression and for converting pulsating dc voltage into relatively smooth dc voltage.

3. Capacitors and inductors are frequency sensitive. This property makes them useful for frequency discrimination.

3.4. Alternating current theory

Commercial quantities of electricity for industry, commerce and domestic use are generated as A.C in large power-stations and distributed around Kenya on the National Grid to the end user. The d.c. electricity has many applications where portability or an emergency stand-by supply is important but for large quantities of power it has to be an A.C. supply because it is so easy to change the voltage levels using a transformer.

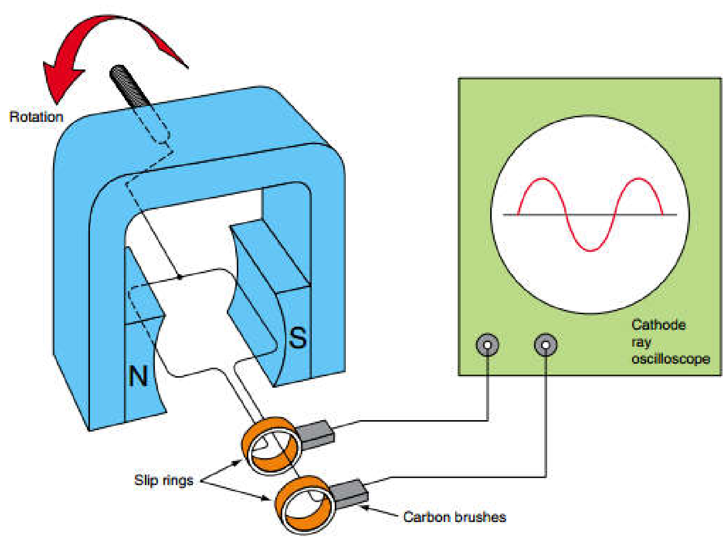


Figure 3.12: Simple A.C. generator or alternator.

Rotating a simple loop of wire or coils of wire between the poles of a magnet, such as that shown simplified in Fig. 3.9, will cut the north–south lines of magnetic flux and induce an a.c. voltage in the loop or coils of wire as shown by the display on a cathode ray oscilloscope. This is an a.c. supply, an alternating current supply. The basic principle of the a.c. supply generated in a power-station is exactly the same as Fig. 3.12 except that powerful electromagnets are used and the power for rotation comes from a steam turbine.

Resistance

In any circuit, resistance is defined as opposition to current flow. From Ohm's law:

$$R = \frac{V_R}{I_R} (\Omega)$$

However, in an a.c. circuit, resistance is only part of the opposition to current flow. The inductance and capacitance of an a.c. circuit also cause an opposition to current flow, which we call *reactance*.

Inductive reactance (X_L) is the opposition to an a.c. current in an inductive circuit. It causes the current in the circuit to lag behind the applied voltage, as shown in Fig. 3.10. It is given by the formula:

$$X_L = 2\pi f L (\Omega)$$

Where

π = 3.142 (a constant)

f = the frequency of the supply

L = the inductance of the circuit or by

$$X_L = \frac{V_L}{I_L}$$

Capacitive reactance (X_C) is the opposition to an a.c. current in a capacitive circuit. It causes the current in the circuit to lead ahead of the voltage, as shown in Fig. 3.10. It is given by the formula:

$$X_C = \frac{1}{2\pi f C} (\Omega)$$

Where π and *f* are defined as before and *C* is the capacitance of the circuit. It can also be expressed as:

$$X_C = \frac{V_C}{I_C}$$

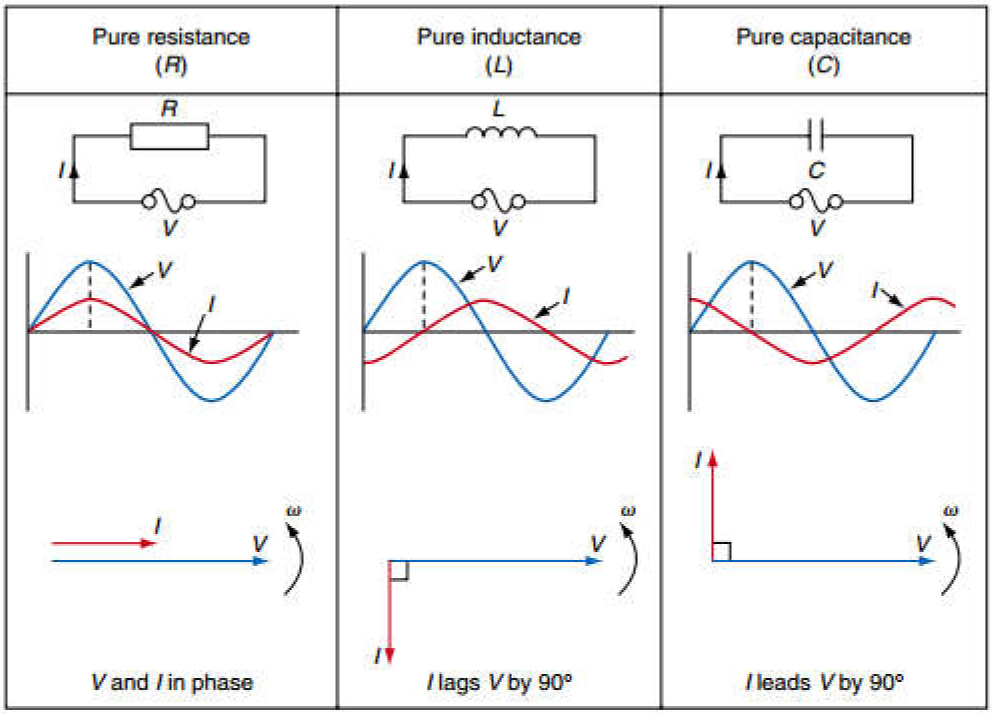


Figure 3.13: Voltage and current relationships in resistive, capacitive and inductive circuits

Example 3.2

Calculate the reactance of a 150μF capacitor and a 0.05 H inductor if they were separately connected to the 50 Hz mains supply;

For capacitive reactance:

$$X_C = \frac{1}{2\pi f C}(\Omega)$$

Where *f* = 50 Hz and *C* = 150μF = 150 X 10^{-6}F

$$X_C = \frac{1}{2 \times 3.142 \times 50Hz \times 150 \times 10^{-6}F} = 21.2\Omega$$

For inductive reactance:

$$X_L = 2\pi f L(\Omega)$$

Where f = 50 Hz and L = 0.05 H

$$X_L = 2 \times 3.142 \times 50Hz \times 0.05H = 15.7\Omega$$

Impedance

The total opposition to current flow in an a.c. circuit is called **impedance** and given the symbol Z. Thus impedance is the combined opposition to current flow of the resistance, inductive reactance and capacitive reactance of the circuit and can be calculated from the formula:

$$Z = \sqrt{R^2 + X^2}\ (\Omega)$$

or

$$Z = \frac{V_T}{I_T}$$

Example 3.3

Calculate the impedance when a 5Ω resistor is connected in series with a 12Ω inductive reactance.

$$Z = \sqrt{R^2 + X_L^2(\Omega)}$$

$$Z = \sqrt{5^2 + 12^2}(\Omega)$$

$$Z = \sqrt{25 + 144}(\Omega)$$

$$Z = \sqrt{169}$$

$$Z = 13\Omega$$

Example 3.4

Calculate the impedance when a 48Ω resistor is connected in series with a 55Ω capacitive reactance.

$$Z = \sqrt{R^2 + X_C^2}(\Omega)$$

$$Z = \sqrt{48^2 + 55^2}(\Omega)$$

$$Z = \sqrt{2305 + 3025}(\Omega)$$

$$Z = \sqrt{5329}$$

$$Z = 73\Omega$$

Resistance, inductance and capacitance in an a.c. circuit

When a resistor only is connected to an a.c. circuit the current and voltage waveforms remain together, starting and finishing at the same time. We say that the waveforms are *in phase*. When a pure inductor is connected to an a.c. circuit the current lags behind the voltage waveform by an angle of 90°. We say that the current *lags* the voltage by 90°. When a pure capacitor is connected to an a.c. circuit the current *leads* the voltage by an angle of 90°. These various effects can be observed on an oscilloscope, but the circuit diagram, waveform diagram and phasor diagram for each circuit are shown in Fig. 3.13.