LECTURE 5-Design of via Frequency Response-Lead Compensator

5.0 Design of a lead compensator

Lead compensators are used to improve transient response of a system. In designing lead compensators via Bode plots, we want to change the phase diagram, increasing the phase margin to reduce the percent overshoot, and increasing the gain crossover to realize a faster transient response. The lead compensator is given by:

$$G_C(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{S+\frac{1}{\beta T}}$$

Where $\beta<1$

The new gain cross over frequency is given by

$$\omega_{max} = \frac{1}{T\sqrt{\beta}}$$

At this frequency the magnitude of the lead compensator will be

$$|G_C(j\omega_{max})| = \frac{1}{\sqrt{\beta}}$$

And the maximum phase shift of the compensator will be

$$\phi_{max} = sin^{-1}\frac{1-\beta}{1+\beta}$$

Example 5.1

For a unity feedback system with a forward transfer function

$$G(s) = \frac{K}{s(s+50)(s+120)}$$

Design a lead compensator for the system in example one to meet the following specifications: OS%= 20%, Ts=0.2 K_v=50

Solution

1. First of all we need to find the closed loop bandwidth to meet the transient response requirement (OS%=20% & Ts= 0.2s).

For 20%= OS%, ξ=0.456, ***Recall*** $\xi = \frac{-ln(\%M_p/100)}{\sqrt{\pi^2+ln^2(\%M_p/100)}}$

Recall (from lecture 4)

$$w_{WB} = \frac{4}{\xi T_s}\sqrt{(1-2\xi^2)+\sqrt{4\xi^4+4\xi^2+2}}$$

W_{BW} =57.89 rad/sec

$$\phi_M = tan^{-1}\left(\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{1+4\xi^4}}}\right)$$

$$\phi_M = 48.15^0$$

2. We need to find the value of K for the uncompensated system so that this value will satisfy the steady state error requirements.

$$K_V = lim_{s\to 0} sG(s)$$

Thus, k=30000

3. Draw the bode plot for the uncompensated system

Matlab code: copy paste the code in red to Matlab workspace.

n=[300000];d=[1,170,6000,0];sys=tf(n,d);bode(sys);margin(sys);

%uncompensated system

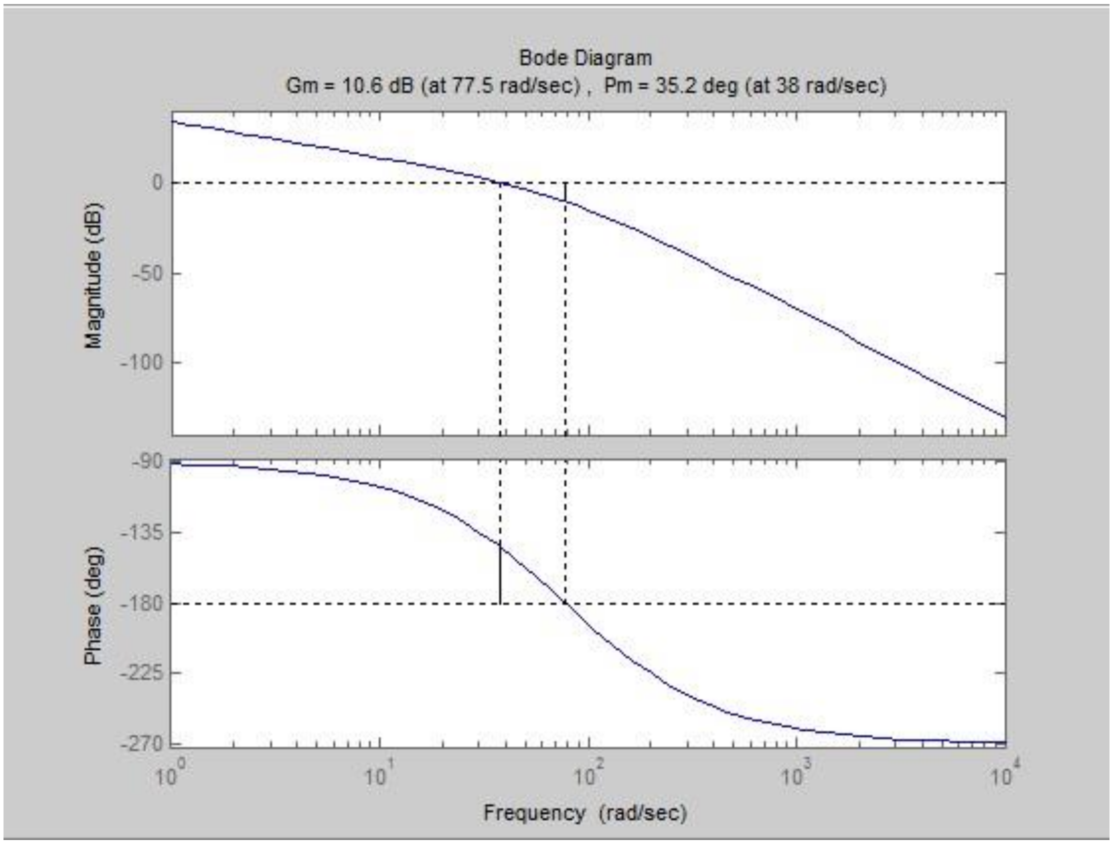
```
n=[300000];

d=[1,170,6000,0];

sys=tf(n,d);

bode(sys);

 margin(sys);
```



From the plot we find that the phase margin of the uncompensated system will be 35.2^0 and the required phase margin is 48.15. So

The maximum phase shift of the compensator is $\phi_{max} = 48.15^0 + 35.2^0 + 10^0 = 23^0$

Where 10 is a correction factor.

And

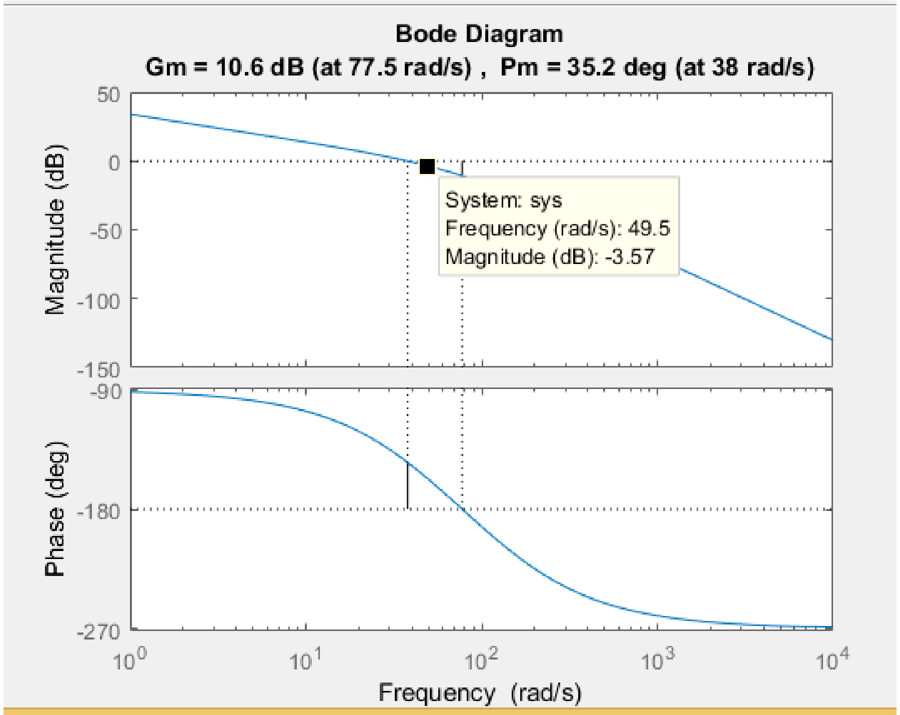
$$\beta = \frac{1 - sin\phi_{max}}{1 + sin\phi_{max}}$$

β=0.438

The compensators magnitude is:$Mag = \frac{1}{\sqrt{\beta}} = 1.509$

Compensator magnitude== $20log1.509 = 3.575dB$

At the gain crossover frequency the magnitude of the compensator is 3.575dB, however the magnitude of the compensated system should be 0dB at this point so the magnitude of the uncompensated system at this point should be -3.575 dB. We find that the gain crossover frequency is (= 49.4 rad/sec) ***try to move the magnitude curve to -3.57)***



Now we will find the zero and pole of the compensator. Note that the compensator should have unity gain in order to keep the steady state requirements as required.

$$\omega_{max} = \frac{1}{T\sqrt{\beta}}$$

Thus T=0.03058, since, ω_{max}*=49.4 rad/sec, &* β*=0.438*

$$G_{C(s)} = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

So the compensator will be,

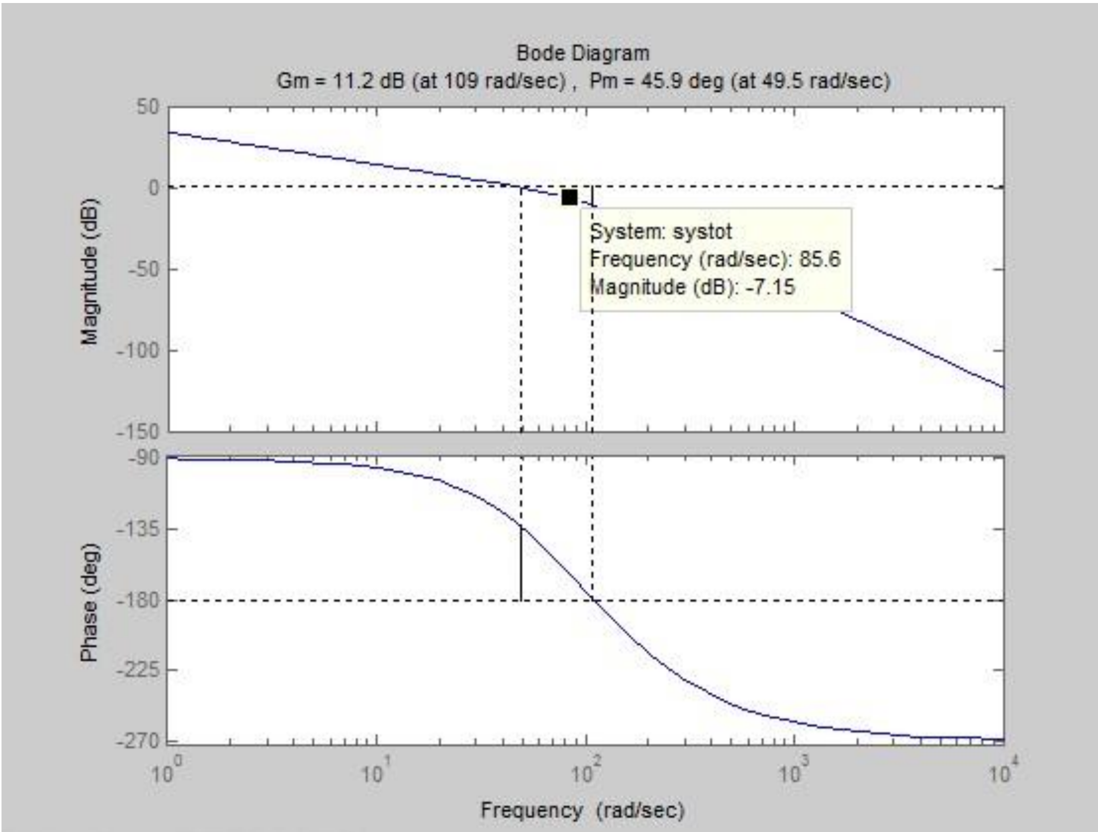
$$G_C(s) = 2.27\frac{s+32.73}{s+74.55}$$

Now we will draw the bode plot of the compensated system

```
nc=[1,32.73];
dc=[1,74.55];
compensator=2.27*tf(nc,dc);

systot=sys*compensator;
bode(systot);
margin(systot);
```

Transfer function:
2.27s + 74.3
----
s + 74.55



Note at -7dB the frequency is 85.6 rad/sec which is greater than the required bandwidth so we expect our design to be correct.

Finally we will plot the step response of the closed-loop system and make sure our design is correct.

Step response code ;(copy paste this code to Matlab workspace)

```
n=[300000];
d=[1,170,6000,0];
sys=tf(n,d);
bode(sys);
 margin(sys);
```

```
nc=[1,32.73];
dc=[1,74.55];
compensator=2.27*tf(nc,dc);

systot=sys*compensator;
bode(systot);
margin(systot);

Sysc= feedback(systot,1);
step(Sysc);
```