LECTURE 2: PD, PID, and Feedback Compensation.

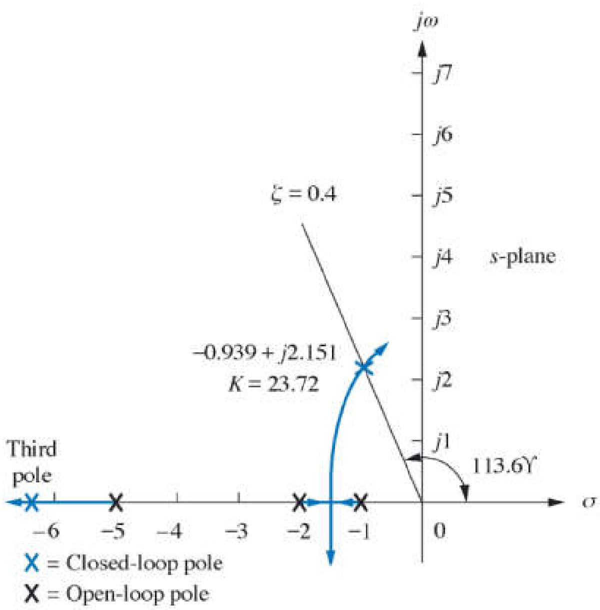
2.1 Ideal Derivative Compensation (PD)

Generally, we want to speed up the transient response (decrease Ts and Tp). If we are lucky then a system's desired transient response lies on its Root Locus (RL). However, if no point on the RL corresponds to the desired transient response then we must compensate the system. A derivative compensator modifies the RL to go through the desired point.
A derivative compensator adds a zero to the forward path.

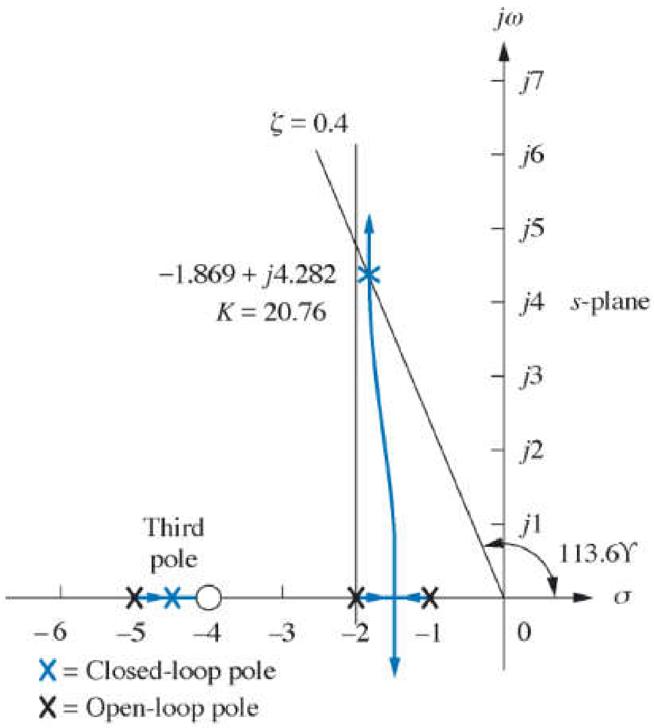
$$G_c(s) = S + Z_c$$

Notice that this transfer function is the sum of a differentiator and a pure gain. Thus, we refer to its use as PD control (proportional + derivative).

We consider various settings for Zc when compensating the system with the following RL:

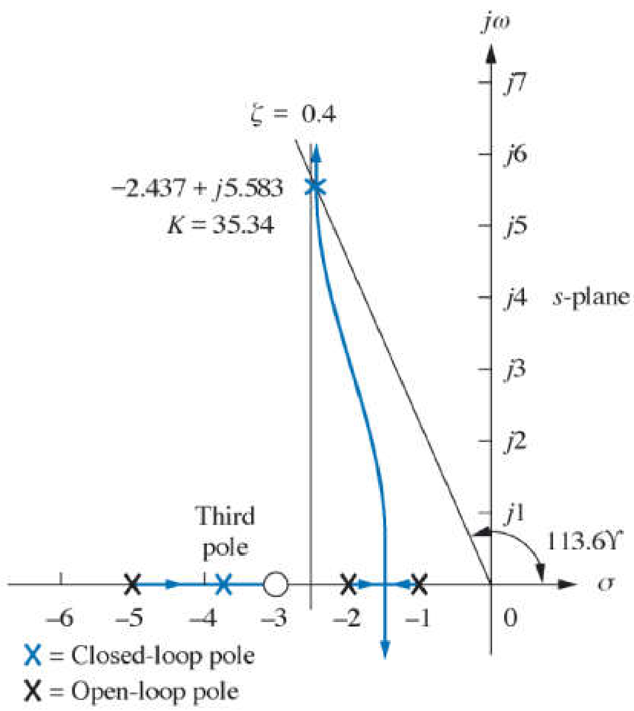


$Zc = -4$



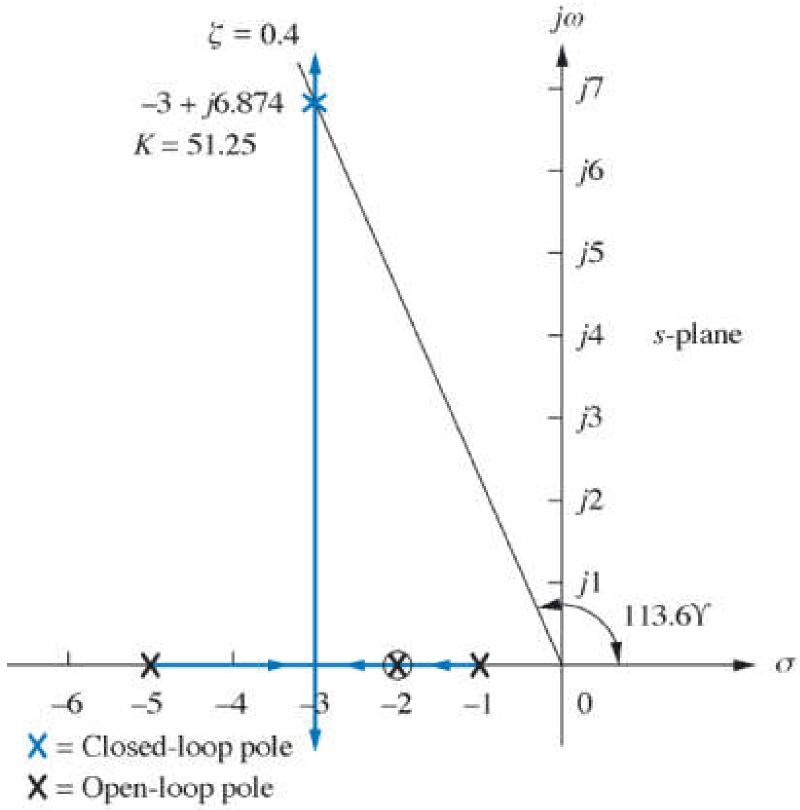
jω
j7
j6
j5
j4
j3
j2
j1
ζ = 0.4
−1.869 + j4.282
K = 20.76
s-plane
Third
pole
113.6Υ
σ
−6
−5
−4
−3
−2
−1
0
= Closed-loop pole
= Open-loop pole

$Zc = -3$

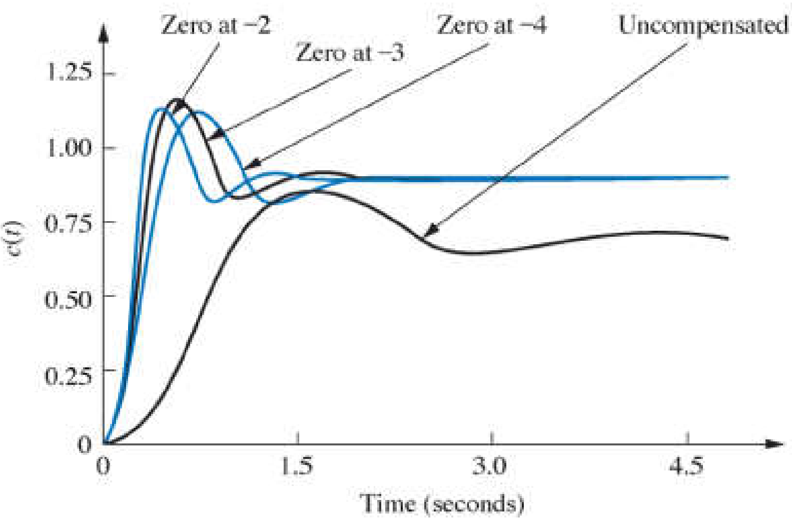


jω
j7
j6
j5
j4
j3
j2
j1
ζ = 0.4
−2.437 + j5.583
K = 35.34
s-plane
Third
pole
113.6Υ
σ
−6
−5
−4
−3
−2
−1
0
= Closed-loop pole
= Open-loop pole

Zc = -2

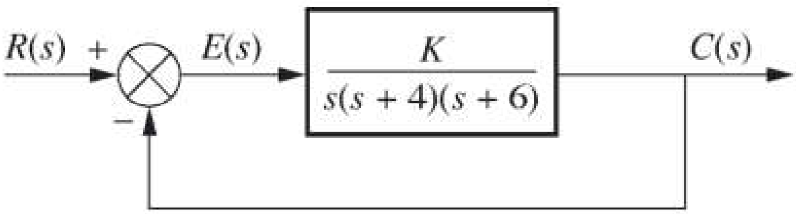


As the zero is moved, we get changes in Ts and Tp. In this case, when the zero is moved to -2 we get the fastest response. All the while, we are maintaining %OS.

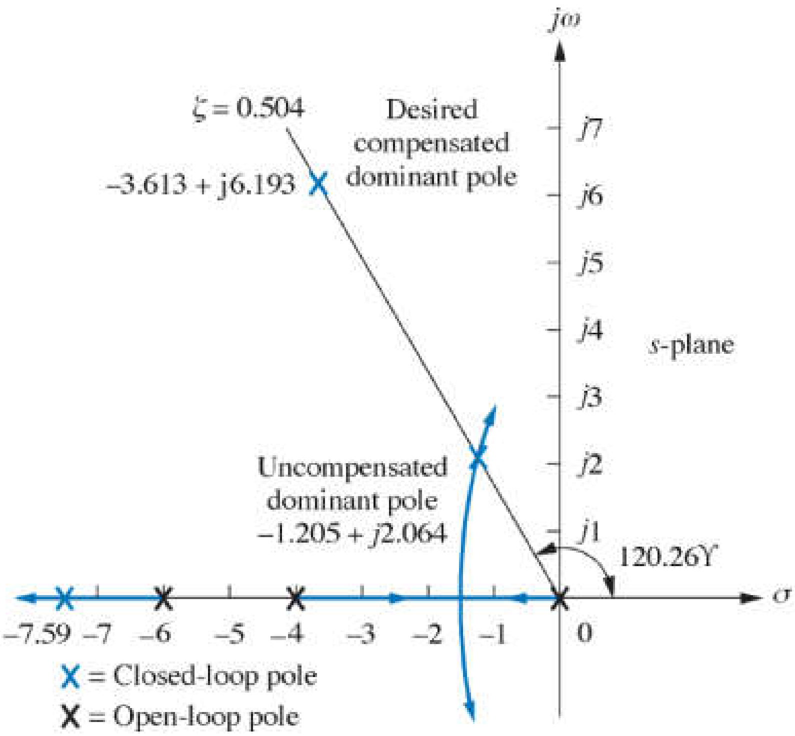


We show how to best place the zero by example...

E.g. Design an ideal derivative compensator for the following system. The ideal transient response has 16% overshoot and a threefold reduction in Ts.



The RL for the uncompensated system:

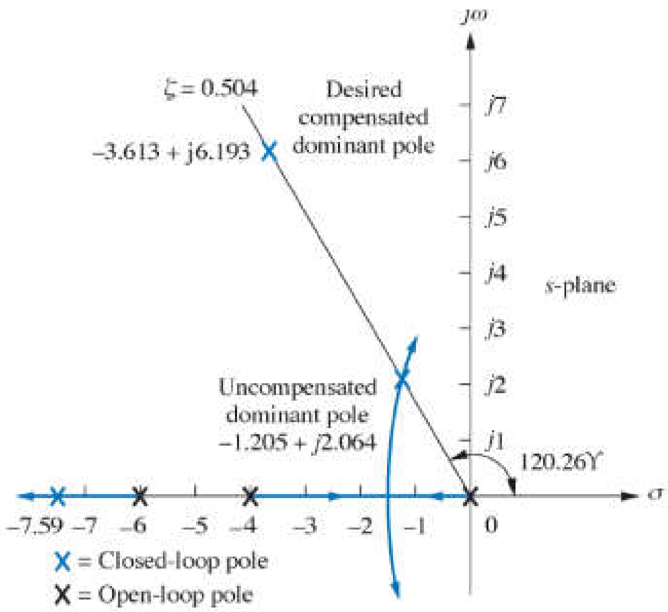


$$T_S = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320$$

The angle made with the positive real-axis must be the same as before (120.26^0) to maintain 16% overshoot. Therefore, we can determine the imaginary part by trigonometry.

$$\tan(180^0 - 120.26^0 = \frac{\omega_d}{3.613}$$

$$\omega_d = 3.613\tan(180^0 - 120.26^0) = 6.193$$

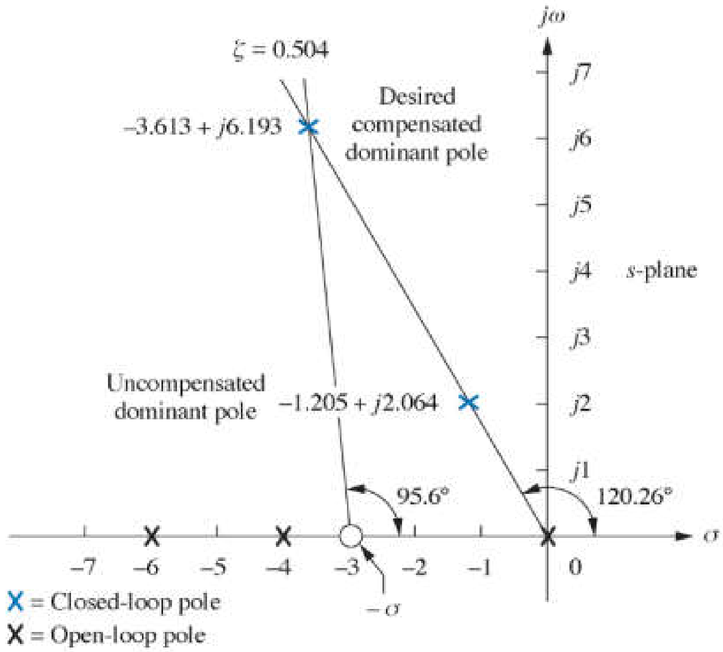


We must now solve for the zero that will place the desired point on the new RL. At the desired point, the sum of angles from the open-loop poles is -275.6^0. To achieve a point on the RL we require a zero positioned so that the sum of angles equals an odd multiple of 180^0

$$-275.6^0 + \theta_{Z_c} = -180^0$$

$$\theta_{Z_c} = 95.6^0$$

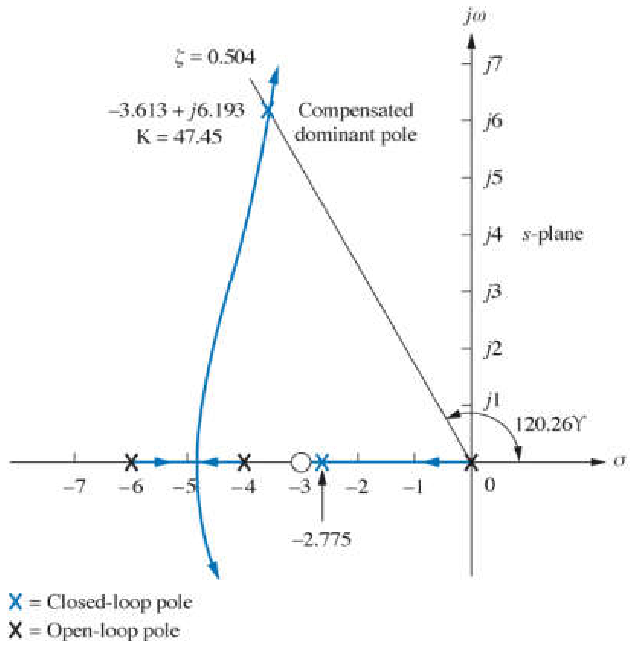
What is the coordinate of a zero that makes an angle of 95:6^0 with the desired complex pole at -3:613 + j6:193?



$$\tan(180^0 - 95.6^0) = \frac{6.193}{3.613 - \sigma}$$

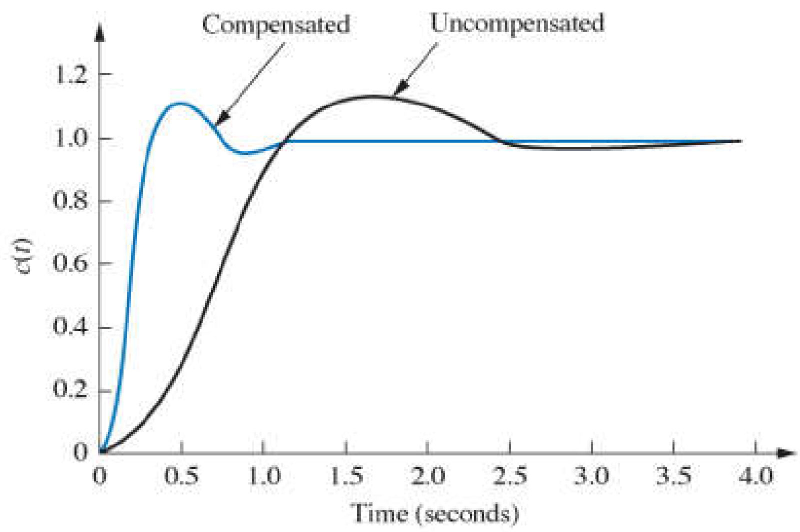
$$\sigma_d = 3.006$$

The RL for the compensated system is as follows:

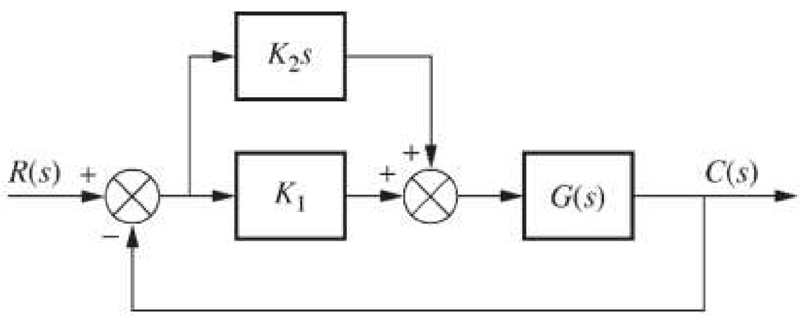


Notice that the 2*nd*-order approximation is not as good for the compensated system. We can determine from simulation that the following quantities differ from their ideal values:

	Ideal	Simulated
%OS	16	11.8
T_S	1.107	1.2
T_P	0.507	0.5



A PD controller can be implemented in a similar manner to the PI controller by placing the proportional and derivative compensators in parallel:



The overall compensator transfer function is as follows:

$$G_c(s) = K_2 s + K_1 = K_2(s + \frac{K_1}{K_2})$$

2.2 Lead Compensation

An ideal derivative compensator has two main disadvantages:

- Differentiation tends to enhance high-frequency noise.
- Implementing a differentiator requires an active circuit.

A lead compensator is, roughly speaking, an approximation to an ideal derivative compensator that can be implemented with a passive circuit. Its transfer function is as follows:

$$K = \frac{s + Z_c}{s + p_c}$$

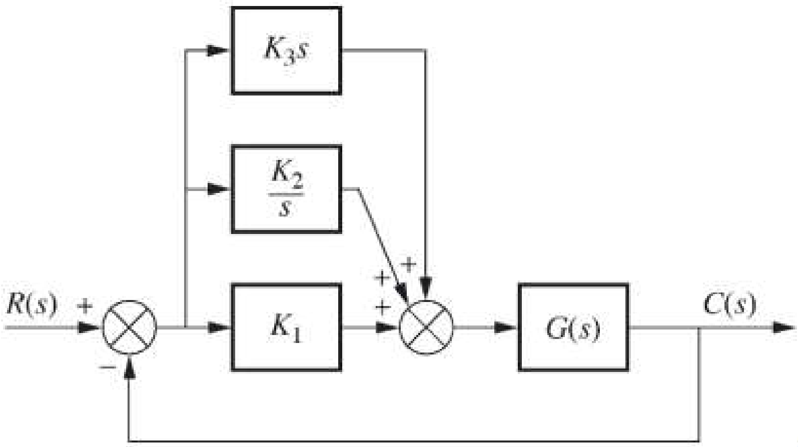
The RL design technique for lead compensators is rather ambiguous; therefore we will not cover it. The frequency response technique (covered later) is more definitive.

2.3 PID Controller Design

A PID controller utilizes PI and PD control together to address both steady-state error and transient response. There are two ways to proceed:

- Design for transient response, then design for steady-state error.
  Con: May slightly decrease response speed when designing for steady-state error.
- Design for steady-state error, then design for transient response
  Con: May increase (or possibly decrease) steady state error when designing for transient response.

We choose to design for transient response first.

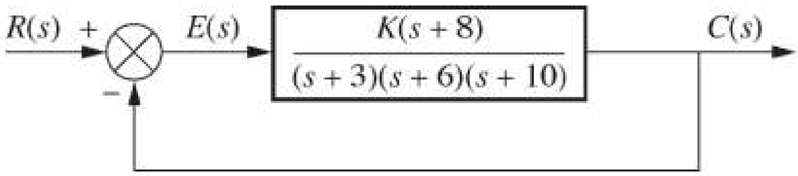


The transfer function for a PID controller is as follows:

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

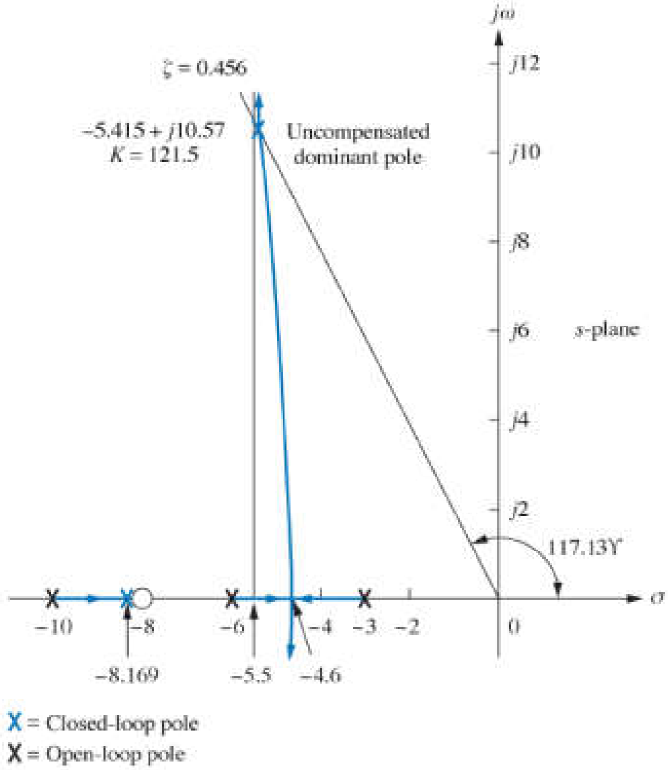
Notice that this function has two zeros and one pole. The location of one zero will come from the transient response design; the other zero will come from the steady-state error design.

E.g. Design a PID controller for the following system which reduces Tp by two thirds, has 20% overshoot, and zero steady-state error for a step input.



First, consider the RL for the uncompensated system at 20%overshoot...

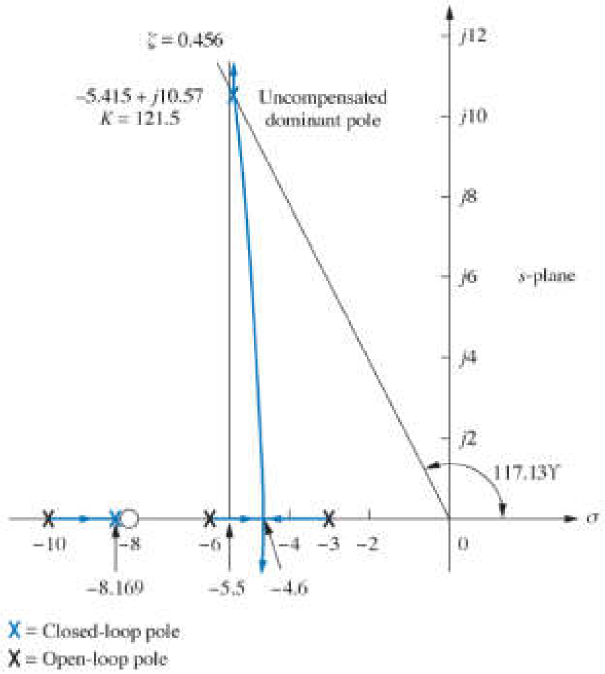
We search to find the current operating point with 20% overshoot:



At this point, $T_p = \frac{\pi}{\omega_d} = 0.297$.

We desire, $T_p = \left(\frac{2}{3}\right) 0.297 = 0.198$

$$\omega_d = \frac{\pi}{0.198} = 15.87$$



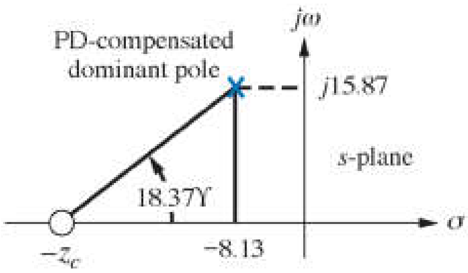
$$\omega_d = 15.87$$

We can then determine the real part of the complex pole by trigonometry:

$$\tan(180^0 - 117.13^0) = \frac{15.87}{\sigma_d}$$

$$\sigma_d = \frac{15.87}{\tan(180^0 - 117.13^0)} = 8.13$$

We must now determine the location of the PD compensator's zero such that this pole lies on the new RL. The current angular sum at -8.13 + j15.87 is -198.37^0. Therefore, the angle that the zero makes with the real-axis should be 18.37^0

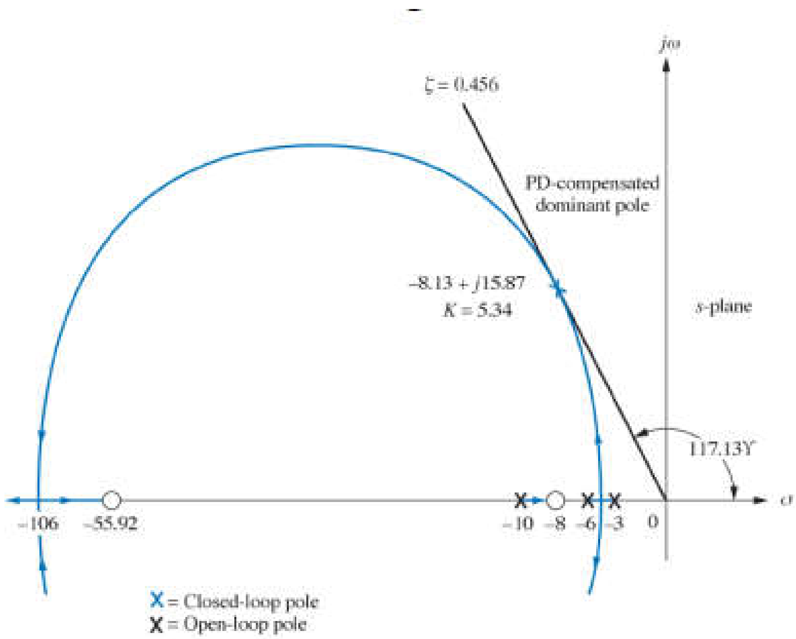


$$tan18.37^0 = \frac{15.87}{Z_c - 8.13}$$

Thus, the location of this zero is at 55.92. The transfer function for the PD-compensated system is,

$$G_{PD}(s) = s + 55.92$$

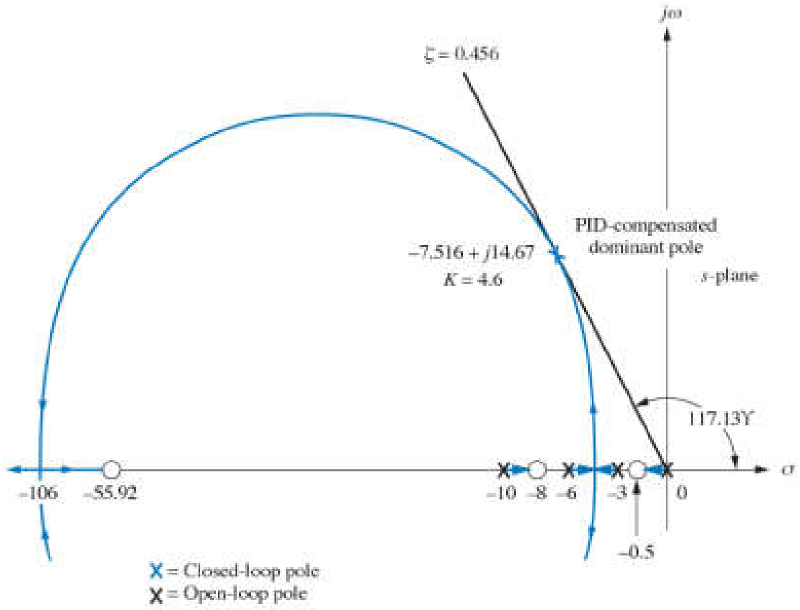
This is the RL for the PD-compensated system. Searching along the zeta = 0:456 line we find the gain is 5.34.



We now compensate this system for steady-state error by adding a pole at the origin and a nearby zero:

$$G_{PI}(s) = \frac{s + 0.5}{s}$$

The following is the RL for the PID-compensated system:



We must search again along the zeta = 0:456 line to find that the gain at the desired operating point is 4.6.

We should now determine the appropriate constants of the PID compensator. The compensator will subsume the gain K that is 4.6. We added a zero at -55.92 for the PD component, and a pole at the origin and a zero at -0.5 for the PI component:

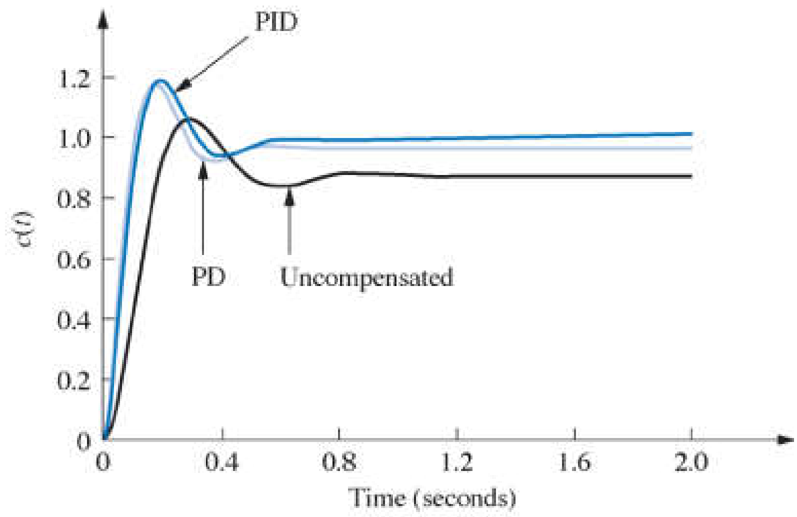
$$G_{PID}(s) = \frac{4.6(s+55.92)(s+0.5)}{s} = \frac{4.6(s^2+56.421s+27.96)}{s}$$

Recall the general form:

$$G_C(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

Hence $K_3 = 4.6$, $K_1 = 259.5$, and $K_2 = 128.6$

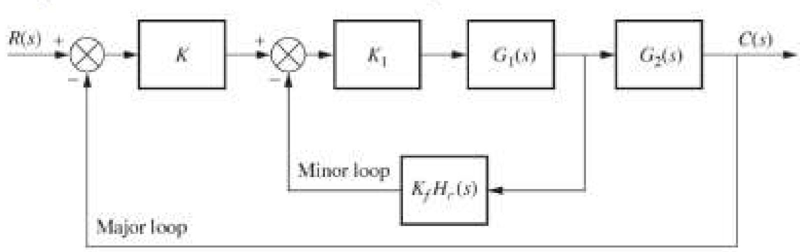
The system's step response shows both the improvement in speed and in reduction of steady-state error:



Since the second-order approximation is no longer valid, it is important to simulate the response to verify that requirements are met. In this case, the desired reduction in Tp of 2/3 was not achieved (uncompensated: 0.297, PID-compensated 0.214). If this is deemed significant, we could re-design the PD component, for a greater than 2/3 reduction in Tp. Alternately, we could move the PI component's zero further from the origin to yield a faster response.

2.4 Feedback Compensation

We have focused on the addition of compensators in the forward path. It is also possible to add compensators in the feedback path:

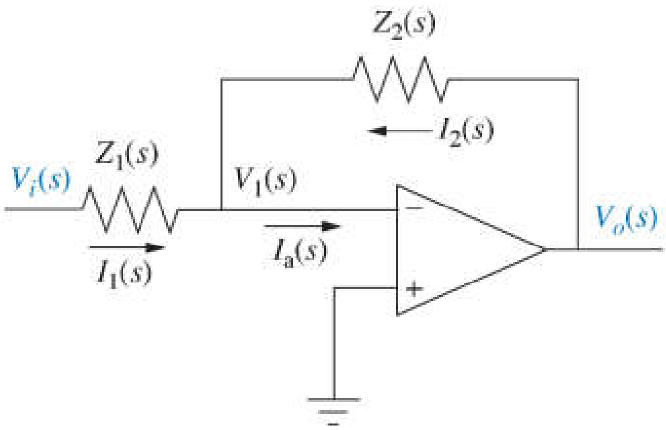


Feedback compensators can yield faster responses than cascade compensators. They also tend to require less amplification since the compensator's input comes from the high-power output of the system, rather than from the low-lower actuating signal. Reduced amplification is preferred in noisy systems where we want to avoid amplifying the noise.

Design techniques for feedback compensators are related to the design techniques for cascade compensators.

2.5 .Physical Realization of Compensation

Utilizing op-amps, we can implement all of the compensators studied. Recall the circuit for an inverting amplifier:



The transfer function for this circuit is:

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

We can achieve a great variety of transfer functions by inserting different components for these impedances.

We can implement lag and lead compensators with both op-amps and with passive circuits.

E.g., recall the transfer function for our example PID compensator.

$$G_C(s) = \frac{K_3(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3})}{s}$$

$$= K_3 s + \frac{K_1}{K_3} + \frac{K_2}{K_3}\frac{1}{s}$$

$$= 4.6s + 56.42 + 27.96\frac{1}{s}$$

We can relate this to the transfer function for a PID controller on,

$$G_c(s) = \left(\frac{R_2}{R_1} + \frac{C_1}{C_2}\right) + R_2 C_1 s + \frac{1}{R_1 C_2}\frac{1}{s}$$

We can establish three equations in four unknowns (R_1, R_2, C_1, and C_2). Choosing an arbitrary value for one component, we can then solve for the other three.