LECTURE 1: Compensation of feedback control systems

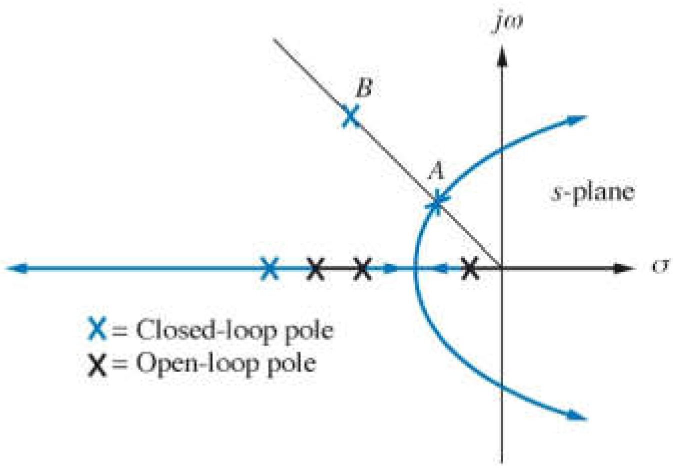
1.1 Introduction.

Fundamentally, we wish to improve the performance of our systems in terms of the principal control systems design criteria:

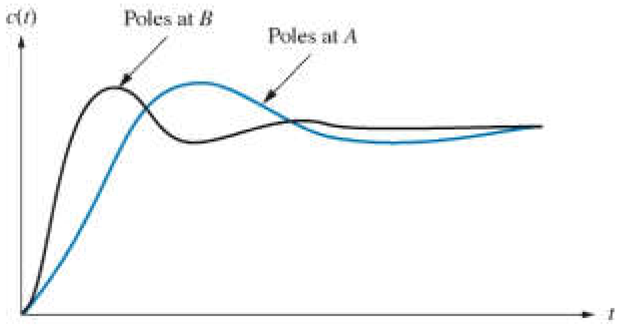
- Transient response,
- Steady-state error and
- Stability.

We saw in the Steady-State Error unit that the addition of integration (1/s trans. function) into the forward path could reduce finite SS error to zero. Here we discuss in more detail how to achieve this.

We also discuss how to improve transient performance. With a given system, the only way we have to adjust transient performance is to find a suitable point on the RL (Root Locus). Assume we have found point A on the RL below which gives us the desired %OS.

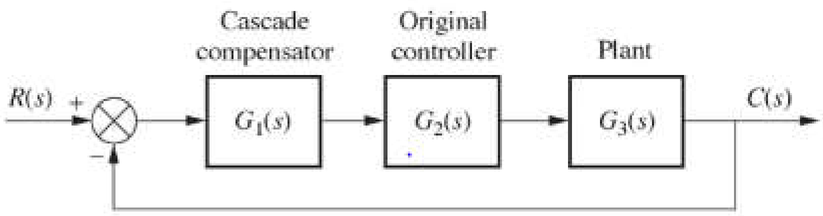


However, while we are happy with the damping ratio ζ and %OS at A we would prefer the reduced settling time at B.

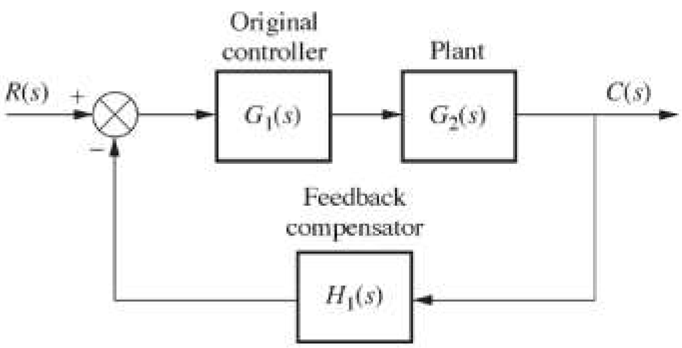


Since B is not on the RL it is not achievable by adjusting gain. Instead, we must compensate the system with additional poles or zeros so that the new system's RL goes through B.

Compensators can be added to a system to improve either steady-state error or transient response. They can either be cascaded with the original controller and plant or added to the feedback path.



Cascade compensator.



Feedback compensator.

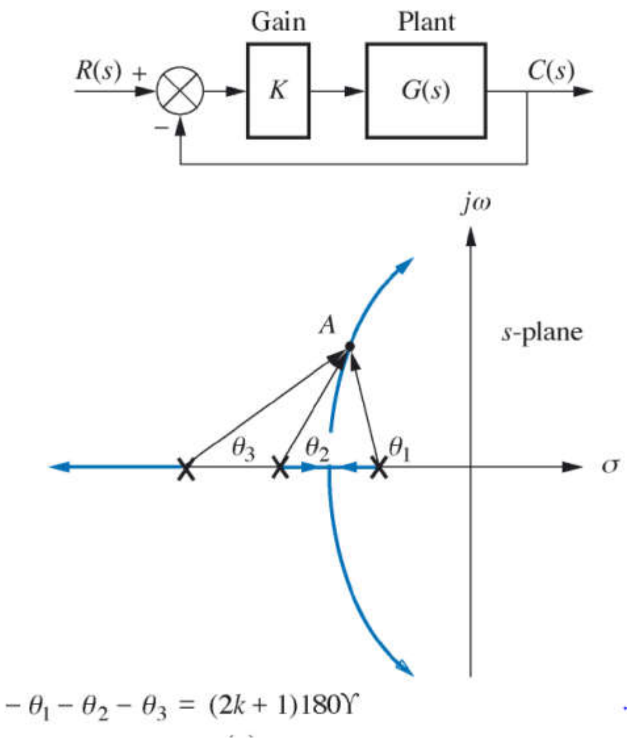
If a compensator involves pure integration or pure differentiation, we refer to it as an ideal compensator. Ideal compensators must be built with active amplifiers. Non-ideal compensators can be constructed using passive components.

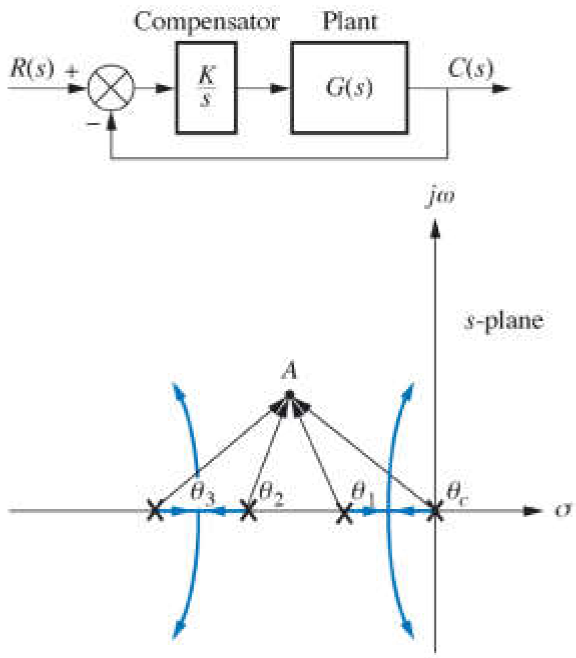
1.2. Ideal Integral Compensation (PI)

Steady-state error can be improved by placement of an open-loop pole at the origin. This increases the system type. Ideally, we would like to add such a zero without affecting the transient response.

This allows transient response to be compensated for separately.

Assume we begin with the system below on the left which is operating at some point A which yields a desirable transient response.

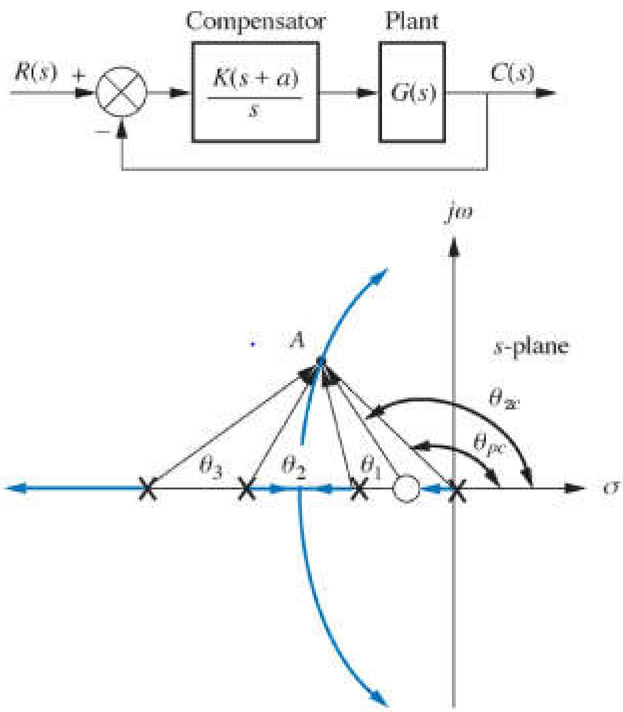




$$-\theta_1 - \theta_2 - \theta_3 - \theta_c \neq (2k+1)180^\circ$$

To improve steady-state error a compensator is added which places a pole at the origin. However, the point A is no longer on the RL.

We can achieve the desired steady-state error while maintaining approximately the same transient response if we also add a zero close to the origin.

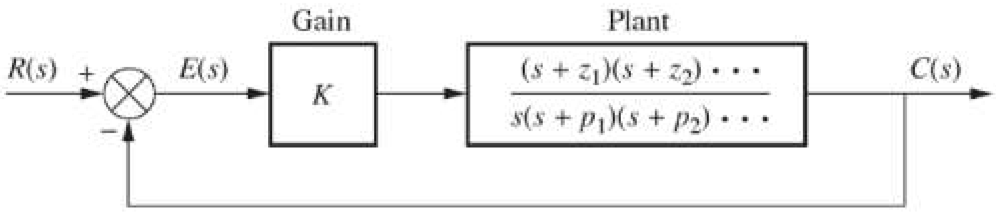


$$-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k+1)180^\circ$$

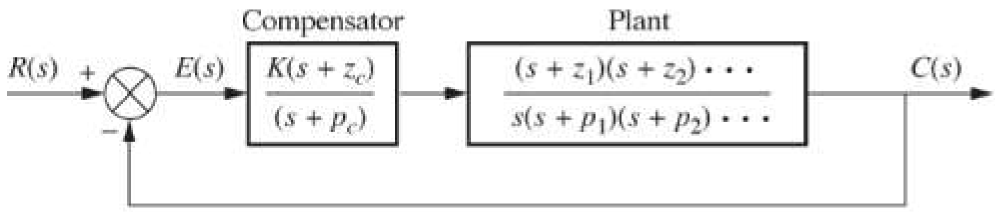
If the zero is close enough to the pole then θpc ≈ θzc which means they effectively cancel out, leaving A still on the RL. The required gain will be similar to the original system, since the ratio of the magnitudes of the added pole to the added zero is approx. 1

1.3. Lag Compensation

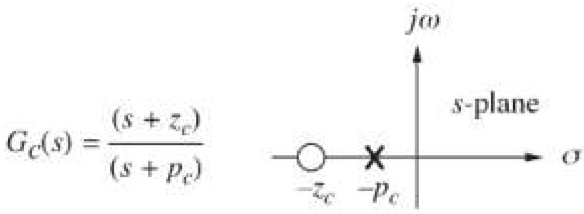
Ideal integral compensation requires an active circuit for implementation. If this is not desirable then instead of placing a pole directly at the origin we can try placing it nearby. Of course, we also place a zero nearby to minimize the impact on transient response. This is known as lag compensation.



(a)



(b)



(c)

The static error constant for the system is,

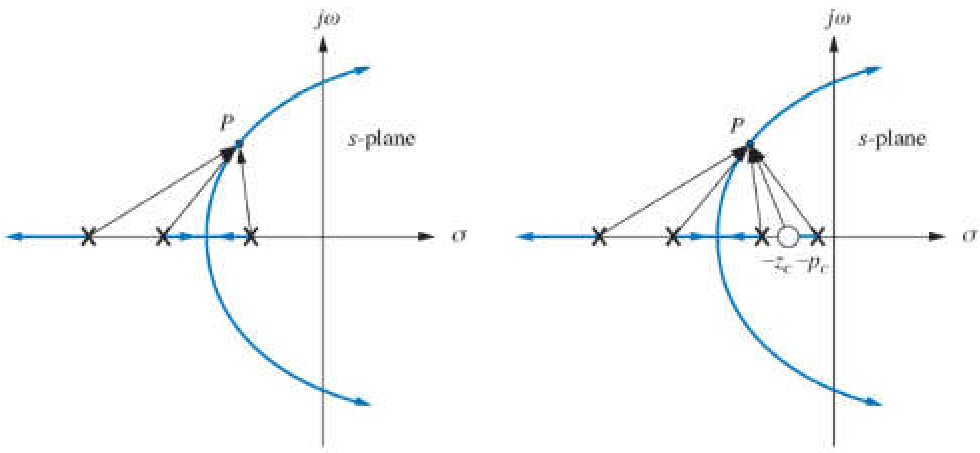
$$K_v = \frac{Kz_1z_2\cdots}{p_1p_2\cdots}$$

With the lag compensator the static error constant becomes

$$\begin{aligned} K_{v_c} &= \frac{Kz_cz_1z_2\cdots}{p_cp_1p_2\cdots} \\ &= \frac{z_c}{p_c}K_v \end{aligned}$$

Thus, we get an increase in static error constant (decrease in $e_{ramp}(\infty)$) when Zc/Pc is large. Are we free to choose how we make this ratio large?

No. If we want to maintain the same transient response, we should position the zero and pole close to each other so that the system's transient response is relatively unaffected.



To make Zc=Pc a large number our only option is tSo move both close to the origin.