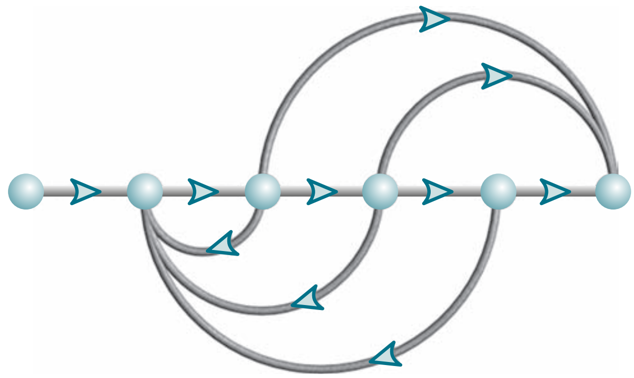
Design via Frequency Response

11



Chapter Learning Outcomes

After completing this chapter the student will be able to:

- Use frequency response techniques to adjust the gain to meet a transient response specification (Sections 11.1–11.2)
- Use frequency response techniques to design cascade compensators to improve the steady-state error (Section 11.3)
- Use frequency response techniques to design cascade compensators to improve the transient response (Section 11.4)
- Use frequency response techniques to design cascade compensators to improve both the steady-state error and the transient response (Section 11.5)

Case Study Learning Outcomes

You will be able to demonstrate your knowledge of the chapter objectives with case studies as follows:

- Given the antenna azimuth position control system shown on the front endpapers, you will be able to use frequency response techniques to design the gain to meet a transient response specification.
- Given the antenna azimuth position control system shown on the front endpapers, you will be able to use frequency response techniques to design a cascade compensator to meet both transient and steady-state error specifications.

11.1 Introduction

In Chapter 8, we designed the transient response of a control system by adjusting the gain along the root locus. The design process consisted of finding the transient response specification on the root locus, setting the gain accordingly, and settling for the resulting steady-state error. The disadvantage of design by gain adjustment is that only the transient response and steady-state error represented by points along the root locus are available.

In order to meet transient response specifications represented by points not on the root locus and, independently, steady-state error requirements, we designed cascade compensators in Chapter 9. In this chapter, we use Bode plots to parallel the root locus design process from Chapters 8 and 9.

Let us begin by drawing some general comparisons between root locus and frequency response design.

Stability and transient response design via gain adjustment. Frequency response design methods, unlike root locus methods, can be implemented conveniently without a computer or other tool except for testing the design. We can easily draw Bode plots using asymptotic approximations and read the gain from the plots. Root locus requires repeated trials to find the desired design point from which the gain can be obtained. For example, in designing gain to meet a percent overshoot requirement, root locus requires the search of a radial line for the point where the open-loop transfer function yields an angle of 180°. To evaluate the range of gain for stability, root locus requires a search of the $j\omega$-axis for 180°. Of course, if one uses a computer program, such as MATLAB, the computational disadvantage of root locus vanishes.

Transient response design via cascade compensation. Frequency response methods are not as intuitive as the root locus, and it is something of an art to design cascade compensation with the methods of this chapter. With root locus, we can identify a specific point as having a desired transient response characteristic. We can then design cascade compensation to operate at that point and meet the transient response specifications. In Chapter 10, we learned that phase margin is related to percent overshoot (Eq. (10.73)) and bandwidth is related to both damping ratio and settling time or peak time (Eqs. (10.55) and (10.56)). These equations are rather complicated. When we design cascade compensation using frequency response methods to improve the transient response, we strive to reshape the open-loop transfer function's frequency response to meet both the phase-margin requirement (percent overshoot) and the bandwidth requirement (settling or peak time). There is no easy way to relate all the requirements prior to the reshaping task. Thus, the reshaping of the open-loop transfer function's frequency response can lead to several trials until all transient response requirements are met.

Steady-state error design via cascade compensation. An advantage of using frequency design techniques is the ability to design derivative compensation, such as lead compensation, to speed up the system and at the same time build in a desired steady-state error requirement that can be met by the lead compensator alone. Recall that in using root locus there are an infinite number of possible solutions to the design of a lead compensator. One of the differences between these solutions is the steady-state error. We must make numerous tries to arrive at the solution that yields the required steady-state error performance. With frequency response techniques, we build the steady-state error requirement right into the design of the lead compensator.

You are encouraged to reflect on the advantages and disadvantages of root locus and frequency response techniques as you progress through this chapter. Let us take a closer look at frequency response design.

When designing via frequency response methods, we use the concepts of stability, transient response, and steady-state error that we learned in Chapter 10. First, the Nyquist criterion tells us how to determine if a system is stable. Typically, an open-loop stable system is stable in closed-loop if the open-loop magnitude frequency response has a gain of less than 0 dB at the frequency where the phase frequency response is 180°. Second, percent overshoot is reduced by increasing the phase margin, and the speed of the response is increased by increasing the bandwidth. Finally, steady-state error is improved by increasing the low-frequency magnitude responses, even if the high-frequency magnitude response is attenuated.

These, then, are the basic facts underlying our design for stability, transient response, and steady-state error using frequency response methods, where the Nyquist criterion and the Nyquist diagram compose the underlying theory behind the design process. Thus, even though we use the Bode plots for ease in obtaining the frequency response, the design process can be verified with the Nyquist diagram when questions arise about interpreting the Bode plots. In particular, when the structure of the system is changed with additional compensator poles and zeros, the Nyquist diagram can offer a valuable perspective.

The emphasis in this chapter is on the design of lag, lead, and lag-lead compensation. General design concepts are presented first, followed by step-by-step procedures. These procedures are only suggestions, and you are encouraged to develop other procedures to arrive at the same goals. Although the concepts in general apply to the design of PI, PD, and PID controllers, in the interest of brevity, detailed procedures and examples will not be presented. You are encouraged to extrapolate the concepts and designs covered and apply them to problems involving PI, PD, and PID compensation presented at the end of this chapter. Finally, the compensators developed in this chapter can be implemented with the realizations discussed in Section 9.6.

11.2 Transient Response via Gain Adjustment

Let us begin our discussion of design via frequency response methods by discussing the link between phase margin, transient response, and gain. In Section 10.10, the relationship between damping ratio (equivalently percent overshoot) and phase margin was derived for $G(s) = \omega_n^2/s(s + 2\zeta\omega_n)$. Thus, if we can vary the phase margin, we can vary the percent overshoot. Looking at Figure 11.1, we see that if we desire a phase margin, Φ_M, represented by CD, we would have to raise the magnitude curve by AB. Thus, a simple gain adjustment can be used to design phase margin and, hence, percent overshoot.

We now outline a procedure by which we can determine the gain to meet a percent overshoot requirement using the open-loop frequency response and assuming dominant second-order closed-loop poles.

Design Procedure

1. Draw the Bode magnitude and phase plots for a convenient value of gain.
2. Using Eqs. (4.39) and (10.73), determine the required phase margin from the percent overshoot.

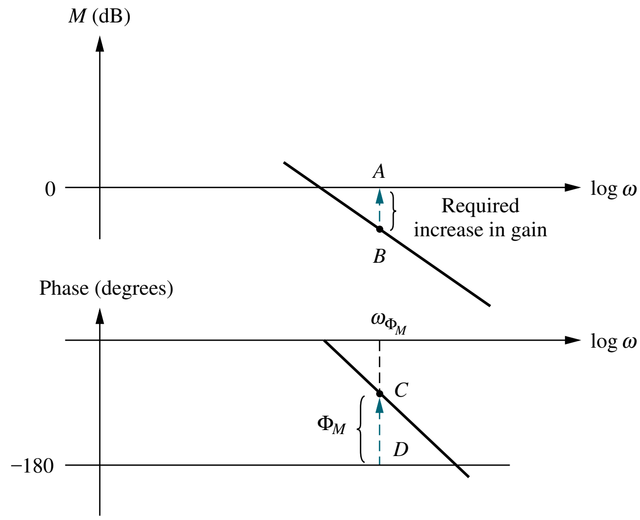


FIGURE 11.1 Bode plots showing gain adjustment for a desired phase margin

3. Find the frequency, ω_{Φ_M}, on the Bode phase diagram that yields the desired phase margin, CD, as shown on Figure 11.1.
4. Change the gain by an amount AB to force the magnitude curve to go through 0 dB at ω_{Φ_M}. The amount of gain adjustment is the additional gain needed to produce the required phase margin.

We now look at an example of designing the gain of a third-order system for percent overshoot.

Example 11.1

Transient Response Design via Gain Adjustment

Design
D

PROBLEM: For the position control system shown in Figure 11.2, find the value of preamplifier gain, K, to yield a 9.5% overshoot in the transient response for a step input. Use only frequency response methods.

SOLUTION: We will now follow the previously described gain adjustment design procedure.

1. Choose $K = 3.6$ to start the magnitude plot at 0 dB at $\omega = 0.1$ in Figure 11.3.
2. Using Eq. (4.39), a 9.5% overshoot implies $\zeta = 0.6$ for the closed-loop dominant poles. Equation (10.73) yields a 59.2° phase margin for a damping ratio of 0.6.

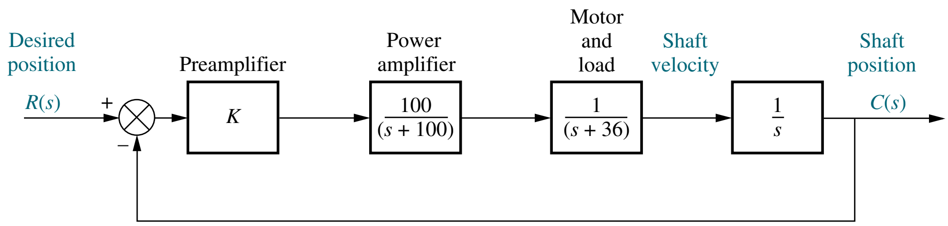


FIGURE 11.2 System for Example 11.1

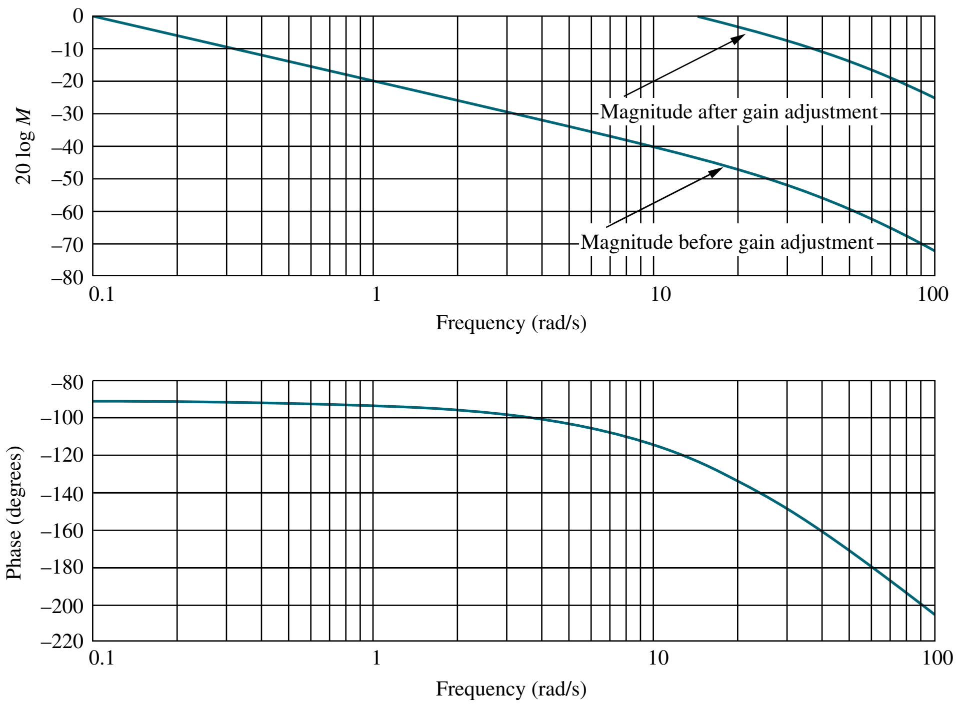


FIGURE 11.3 Bode magnitude and phase plots for Example 11.1

3. Locate on the phase plot the frequency that yields a 59.2° phase margin. This frequency is found where the phase angle is the difference between $-180°$ and 59.2°, or $-120.8°$. The value of the phase-margin frequency is 14.8 rad/s.
4. At a frequency of 14.8 rad/s on the magnitude plot, the gain is found to be -44.2 dB. This magnitude has to be raised to 0 dB to yield the required phase margin. Since the log-magnitude plot was drawn for $K = 3.6$, a 44.2 dB increase, or $K = 3.6 \times 162.2 = 583.9$, would yield the required phase margin for 9.48% overshoot.

The gain-adjusted open-loop transfer function is

$$G(s) = \frac{58{,}390}{s(s+36)(s+100)} \tag{11.1}$$

Table 11.1 summarizes a computer simulation of the gain-compensated system.

TABLE 11.1 Characteristic of gain-compensated system of Example 11.1

Parameter	Proposed specification	Actual value
K_v	—	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	—	14.8 rad/s
Percent overshoot	9.5	10
Peak time	—	0.18 second

MATLAB
ML

Students who are using MATLAB should now run ch11p1 in Appendix B. You will learn how to use MATLAB to design a gain to meet a percent overshoot specification using Bode plots. This exercise solves Example 11.1 using MATLAB.

Skill-Assessment Exercise 11.1

WileyPLUS
WPCS
Control Solutions

PROBLEM: For a unity feedback system with a forward transfer function

$$G(s) = \frac{K}{s(s+50)(s+120)}$$

use frequency response techniques to find the value of gain, K, to yield a closed-loop step response with 20% overshoot.

ANSWER: $K = 194{,}200$

The complete solution is located at www.wiley.com/college/nise.

TryIt 11.1

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.1.

```
pos=20
z=(-log(pos/100))/...
 (sqrt(pi^2+...
 log(pos/100)^2))
Pm=atan(2*z/...
 (sqrt(-2*z^2+...
 sqrt(1+4*z^4))))*...
 (180/pi)
G=zpk([],...
 [0.-50,-120],1)
sisotool
```

In the SISOTOOL Window:

1. Select **Import** . . . in the **File** menu.
2. Click on **G** in the **System Data Window** and click **Browse** . . .
3. In the **Model Import Window** select radio button Workspace and select **G** in **Available Models**. Click Import, then Close.
4. Click Ok in the **System Data Window.**
5. Right-click in the Bode graph area and be sure all selections under **Show** are checked.
6. Grab the stability margin point in the magnitude diagram and raise the magnitude curve until the phase curve shows the phase margin calculated by the program and shown in the MATLAB **Command Window** as Pm.
7. Right-click in the Bode plot area, select **Edit Compensator** . . . and read the gain under **Compensator** in the resulting window.

In this section, we paralleled our work in Chapter 8 with a discussion of transient response design through gain adjustment. In the next three sections, we parallel the root locus compensator design in Chapter 9 and discuss the design of lag, lead, and lag-lead compensation via Bode diagrams.

11.3 Lag Compensation

In Chapter 9, we used the root locus to design lag networks and PI controllers. Recall that these compensators permitted us to design for steady-state error without appreciably affecting the transient response. In this section, we provide a parallel development using the Bode diagrams.

Visualizing Lag Compensation

The function of the lag compensator as seen on Bode diagrams is to (1) improve the static error constant by increasing only the low-frequency gain without any resulting instability, and (2) increase the phase margin of the system to yield the desired transient response. These concepts are illustrated in Figure 11.4.

The uncompensated system is unstable since the gain at 180° is greater than 0 dB. The lag compensator, while not changing the low-frequency gain, does reduce

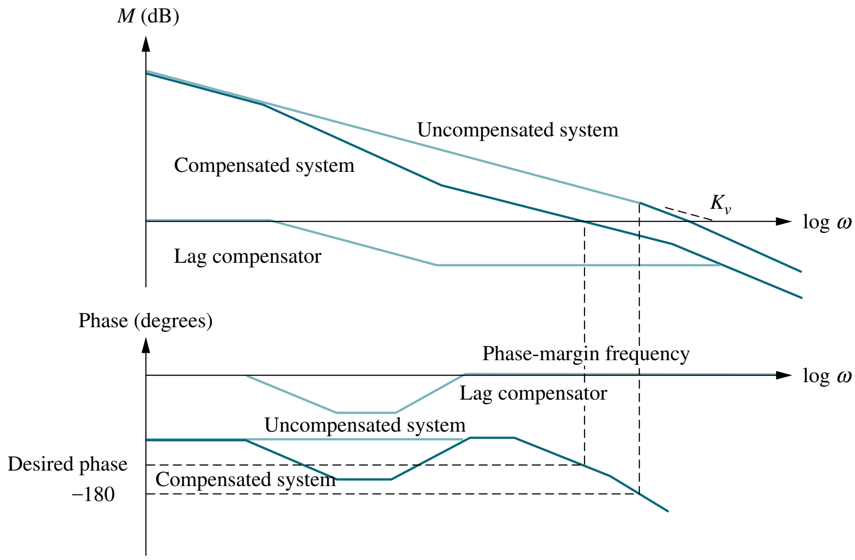


FIGURE 11.4 Visualizing lag compensation

the high-frequency gain.[1] Thus, the low-frequency gain of the system can be made high to yield a large K_v without creating instability. This stabilizing effect of the lag network comes about because the gain at 180° of phase is reduced below 0 dB. Through judicious design, the magnitude curve can be reshaped, as shown in Figure 11.4, to go through 0 dB at the desired phase margin. Thus, both K_v and the desired transient response can be obtained. We now enumerate a design procedure.

Design Procedure

1. Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain.
2. Find the frequency where the phase margin is 5° to 12° greater than the phase margin that yields the desired transient response (*Ogata, 1990*). This step compensates for the fact that the phase of the lag compensator may still contribute anywhere from −5° to − 12° of phase at the phase-margin frequency.
3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through 0 dB at the frequency found in Step 2 as follows: Draw the compensator's high-frequency asymptote to yield 0 dB for the compensated system at the frequency found in Step 2. Thus, if the gain at the frequency found in Step 2 is $20 \log K_{PM}$, then the compensator's high-frequency asymptote will be set at $-20 \log K_{PM}$; select the upper break frequency to be 1 decade below the frequency found in Step 2;[2] select the low-frequency asymptote to be at 0 dB; connect the compensator's high- and low-frequency asymptotes with a −20 dB/decade line to locate the lower break frequency.
4. Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1.

[1] The name *lag compensator* comes from the fact that the typical phase angle response for the compensator, as shown in Figure 11.4, is always negative, or *lagging* in phase angle.

[2] This value of break frequency ensures that there will be only −5° to − 12° phase contribution from the compensator at the frequency found in Step 2.

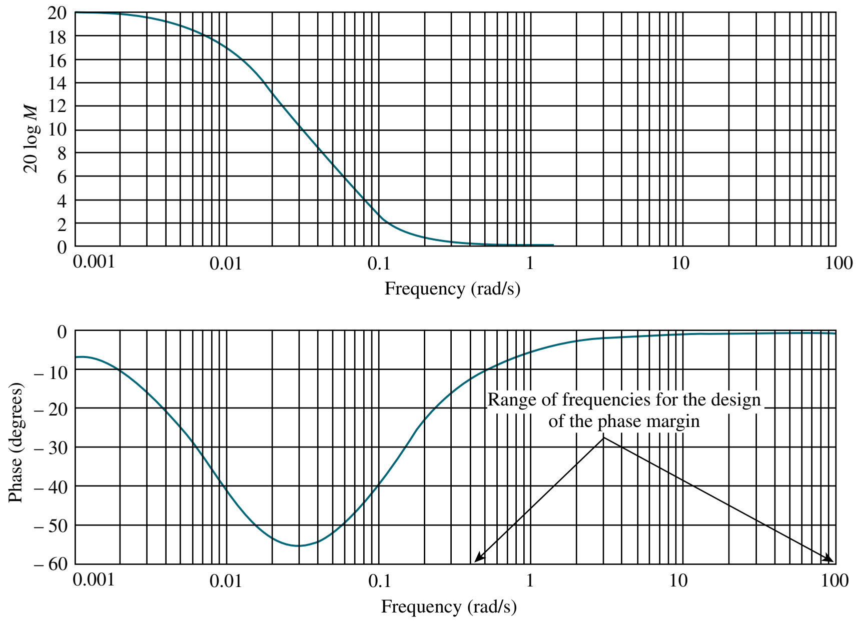


FIGURE 11.5 Frequency response plots of a lag compensator, $G_c(s) = (s + 0.1)/(s + 0.01)$

From these steps, you see that we are relying upon the initial gain setting to meet the steady-state requirements and then relying upon the lag compensator's -20 dB/decade slope to meet the transient response requirement by setting the 0 dB crossing of the magnitude plot.

The transfer function of the lag compensator is

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \tag{11.2}$$

where $\alpha > 1$.

Figure 11.5 shows the frequency response curves for the lag compensator. The range of high frequencies shown in the phase plot is where we will design our phase margin. This region is after the second break frequency of the lag compensator, where we can rely on the attenuation characteristics of the lag network to reduce the total open-loop gain to unity at the phase-margin frequency. Further, in this region the phase response of the compensator will have minimal effect on our design of the phase margin. Since there is still some effect, approximately 5° to 12°, we will add this amount to our phase margin to compensate for the phase response of the lag compensator (see Step 2).

Example 11.2

Lag Compensation Design

Design **D**

PROBLEM: Given the system of Figure 11.2, use Bode diagrams to design a lag compensator to yield a tenfold improvement in steady-state error over the gain-compensated system while keeping the percent overshoot at 9.5%.

SOLUTION: We will follow the previously described lag compensation design procedure.

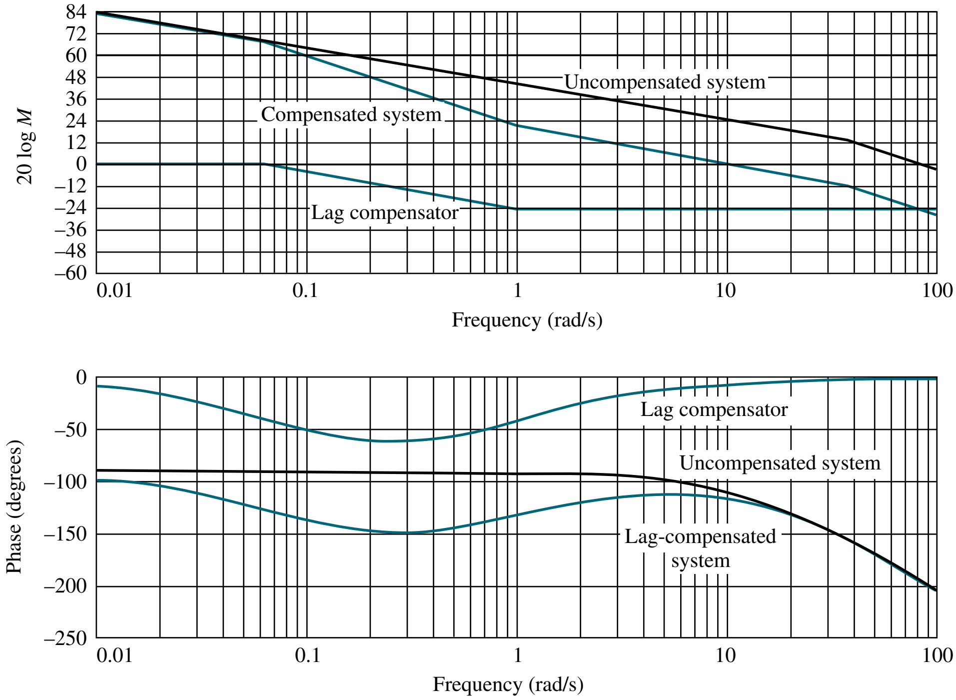


FIGURE 11.6 Bode plots for Example 11.2.

1. From Example 11.1 a gain, K, of 583.9 yields a 9.5% overshoot. Thus, for this system, $K_v = 16.22$. For a tenfold improvement in steady-state error, K_v must increase by a factor of 10, or $K_v = 162.2$. Therefore, the value of K in Figure 11.2 equals 5839, and the open-loop transfer function is

$$G(s) = \frac{583{,}900}{s(s+36)(s+100)} \tag{11.3}$$

The Bode plots for $K = 5839$ are shown in Figure 11.6.

2. The phase margin required for a 9.5% overshoot ($\zeta = 0.6$) is found from Eq. (10.73) to be 59.2°. We increase this value of phase margin by 10° to 69.2° in order to compensate for the phase angle contribution of the lag compensator. Now find the frequency where the phase margin is 69.2°. This frequency occurs at a phase angle of $-180° + 69.2° = -110.8°$ and is 9.8 rad/s. At this frequency, the magnitude plot must go through 0 dB. The magnitude at 9.8 rad/s is now +24 dB (exact, that is, nonasymptotic). Thus, the lag compensator must provide −24 dB attenuation at 9.8 rad/s.

3.&4. We now design the compensator. First draw the high-frequency asymptote at −24 dB. Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency, or 0.98 rad/s. Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a −20 dB/decade line until 0 dB is reached. The compensator must have a dc gain of unity to retain the value of K_v that we have already designed by setting $K = 5839$. The lower break frequency is found to be 0.062 rad/s. Hence, the lag compensator's transfer function is

$$G_c(s) = \frac{0.063(s+0.98)}{(s+0.062)} \tag{11.4}$$

where the gain of the compensator is 0.063 to yield a dc gain of unity.

The compensated system's forward transfer function is thus

$$G(s)G_c(s) = \frac{36{,}786(s+0.98)}{s(s+36)(s+100)(s+0.062)} \tag{11.5}$$

The characteristics of the compensated system, found from a simulation and exact frequency response plots, are summarized in Table 11.2.

TABLE 11.2 Characteristics of the lag-compensated system of Example 11.2

Parameter	Proposed specification	Actual value
K_v	162.2	161.5
Phase margin	59.2°	62°
Phase-margin frequency	—	11 rad/s
Percent overshoot	9.5	10
Peak time	—	0.25 second

MATLAB

ML

Students who are using MATLAB should now run ch11p2 in Appendix B. You will learn how to use MATLAB to design a lag compensator. You will enter the value of gain to meet the steady-state error requirement as well as the desired percent overshoot. MATLAB then designs a lag compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.2 using MATLAB.

Skill-Assessment Exercise 11.2

PROBLEM: Design a lag compensator for the system in Skill-Assessment Exercise 11.1 that will improve the steady-state error tenfold, while still operating with 20% overshoot.

ANSWER:

$$G_{lag}(s) = \frac{0.0691(s+2.04)}{(s+0.141)}; \quad G(s) = \frac{1{,}942{,}000}{s(s+50)(s+120)}$$

The complete solution is at www.wiley.com/college/nise.

TryIt 11.2

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.2.

```
pos=20
Ts=0.2
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2))
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
Wbw=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2))
K=1942000
G=zpk([],[0,-50,-120],K)
sisotool(G,1)
```

(*TryIt continues*)

(*TryIt Continued*)
When the **SISO Design for SISO Design Task Window** appears:

1. Right-click on the Bode plot area and select **Grid.**
2. Note the phase margin shown in the MATLAB **Command Window.**
3. Using the Bode phase plot, estimate the frequency at which the phase margin from Step 2 occurs.
4. On the **SISO Design for SISO Design Task Window toolbar**, click on the red zero.
5. Place the zero of the compensator by clicking on the gain plot at a frequency that is 1/10 that found in Step 3.
6. On the **SISO Design for SISO Design Task Window toolbar**, click on the red pole.
7. Place the pole of the compensator by clicking on the gain plot to the left of the compensator zero.
8. Grab the pole with the mouse and move it until the phase plot shows a P.M. equal to that found in Step 2.
9. Right-click in the Bode plot area and select **Edit Compensator . . .**
10. Read the lag compensator in the **Control and Estimation Tools Manager Window.**

In this section, we showed how to design a lag compensator to improve the steady-state error while keeping the transient response relatively unaffected. We next discuss how to improve the transient response using frequency response methods.

11.4 Lead Compensation

For second-order systems, we derived the relationship between phase margin and percent overshoot as well as the relationship between closed-loop bandwidth and other time-domain specifications, such as settling time, peak time, and rise time. When we designed the lag network to improve the steady-state error, we wanted a minimal effect on the phase diagram in order to yield an imperceptible change in the transient response. However, in designing lead compensators via Bode plots, we want to change the phase diagram, increasing the phase margin to reduce the percent overshoot, and increasing the gain crossover to realize a faster transient response.

Visualizing Lead Compensation

The lead compensator increases the bandwidth by increasing the gain crossover frequency. At the same time, the phase diagram is raised at higher frequencies. The result is a larger phase margin and a higher phase-margin frequency. In the time domain, lower percent overshoots (larger phase margins) with smaller peak times (higher phase-margin frequencies) are the results. The concepts are shown in Figure 11.7.

The uncompensated system has a small phase margin (B) and a low phase-margin frequency (A). Using a phase lead compensator, the phase angle plot (compensated system) is raised for higher frequencies.[3] At the same time, the gain crossover frequency in the magnitude plot is increased from A rad/s to C rad/s. These effects yield a larger phase margin (D), a higher phase-margin frequency (C), and a larger bandwidth.

One advantage of the frequency response technique over the root locus is that we can implement a steady-state error requirement and then design a transient response. This specification of transient response with the constraint of a steady-state error is easier to implement with the frequency response technique than with the root locus. Notice that the initial slope, which determines the steady-state error, is not affected by the design for the transient response.

[3] The name *lead compensator* comes from the fact that the typical phase angle response shown in Figure 11.7 is always positive, or *leading* in phase angle.

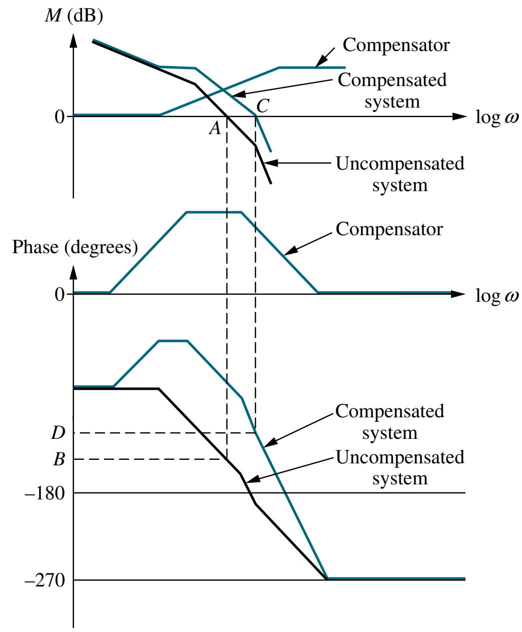


FIGURE 11.7 Visualizing lead compensation

Lead Compensator Frequency Response

Let us first look at the frequency response characteristics of a lead network and derive some valuable relationships that will help us in the design process. Figure 11.8 shows plots of the lead network

$$G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \tag{11.6}$$

for various values of β, where $\beta < 1$. Notice that the peaks of the phase curve vary in maximum angle and in the frequency at which the maximum occurs. The dc gain of the compensator is set to unity with the coefficient $1/\beta$, in order not to change the dc gain designed for the static error constant when the compensator is inserted into the system.

In order to design a lead compensator and change both the phase margin and phase-margin frequency, it is helpful to have an analytical expression for the maximum value of phase and the frequency at which the maximum value of phase occurs, as shown in Figure 11.8.

From Eq. (11.6) the phase angle of the lead compensator, ϕ_c, is

$$\phi_c = \tan^{-1}\omega T - \tan^{-1}\omega\beta T \tag{11.7}$$

Differentiating with respect to ω, we obtain

$$\frac{d\phi_c}{d\omega} = \frac{T}{1+(\omega T)^2} - \frac{\beta T}{1+(\omega\beta T)^2} \tag{11.8}$$

Setting Eq. (11.8) equal to zero, we find that the frequency, ω_{max}, at which the maximum phase angle, ϕ_{max}, occurs is

$$\omega_{max} = \frac{1}{T\sqrt{\beta}} \tag{11.9}$$

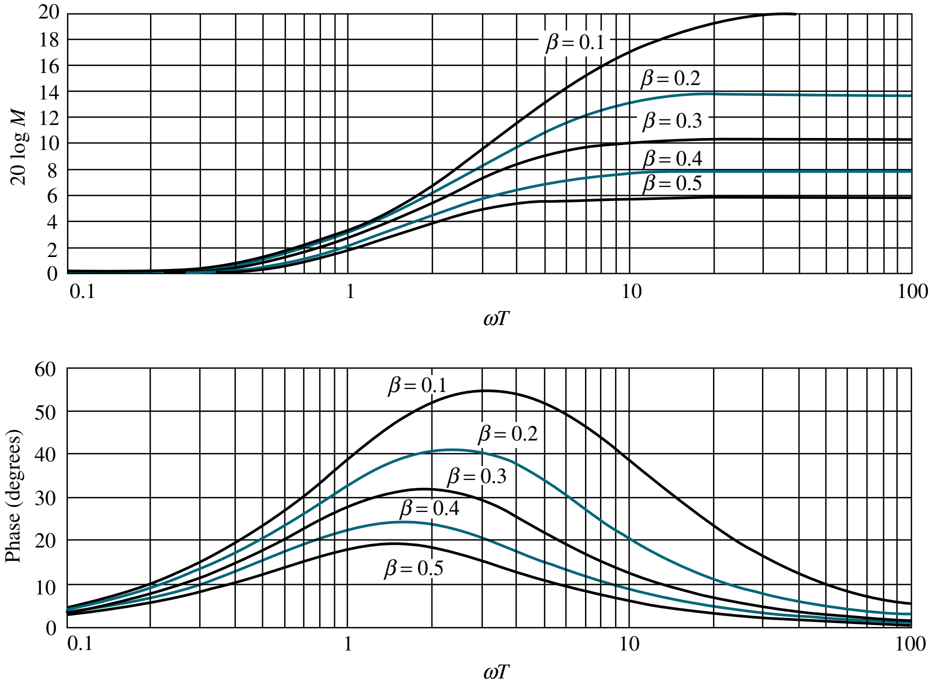


FIGURE 11.8 Frequency response of a lead compensator, $G_c(s) = [1/\beta][(s + 1/T)/(s + 1/\beta T)]$

Substituting Eq. (11.9) into Eq. (11.6) with $s = j\omega_{\max}$,

$$G_c(j\omega_{\max}) = \frac{1}{\beta}\frac{j\omega_{\max} + \dfrac{1}{T}}{j\omega_{\max} + \dfrac{1}{\beta T}} = \frac{j\dfrac{1}{\sqrt{\beta}} + 1}{j\sqrt{\beta} + 1} \tag{11.10}$$

Making use of $\tan(\phi_1 - \phi_2) = (\tan\phi_1 - \tan\phi_2)/(1 + \tan\phi_1 \tan\phi_2)$, the maximum phase shift of the compensator, $\phi_{\max}$, is

$$\phi_{\max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1}\frac{1-\beta}{1+\beta} \tag{11.11}$$

and the compensator's magnitude at $\omega_{\max}$ is

$$|G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}} \tag{11.12}$$

We are now ready to enumerate a design procedure.

Design Procedure

1. Find the closed-loop bandwidth required to meet the settling time, peak time, or rise time requirement (see Eqs. (10.54) through (10.56)).
2. Since the lead compensator has negligible effect at low frequencies, set the gain, K, of the uncompensated system to the value that satisfies the steady-state error requirement.

3. Plot the Bode magnitude and phase diagrams for this value of gain and determine the uncompensated system's phase margin.
4. Find the phase margin to meet the damping ratio or percent overshoot requirement. Then evaluate the additional phase contribution required from the compensator.[4]
5. Determine the value of β (see Eqs. (11.6) and (11.11)) from the lead compensator's required phase contribution.
6. Determine the compensator's magnitude at the peak of the phase curve (Eq. (11.12)).
7. Determine the new phase-margin frequency by finding where the uncompensated system's magnitude curve is the negative of the lead compensator's magnitude at the peak of the compensator's phase curve.
8. Design the lead compensator's break frequencies, using Eqs. (11.6) and (11.9) to find T and the break frequencies.
9. Reset the system gain to compensate for the lead compensator's gain.
10. Check the bandwidth to be sure the speed requirement in Step 1 has been met.
11. Simulate to be sure all requirements are met.
12. Redesign if necessary to meet requirements.

From these steps, we see that we are increasing both the amount of phase margin (improving percent overshoot) and the gain crossover frequency (increasing the speed). Now that we have enumerated a procedure with which we can design a lead compensator to improve the transient response, let us demonstrate.

Example 11.3

Design

D

Lead Compensation Design

PROBLEM: Given the system of Figure 11.2, design a lead compensator to yield a 20% overshoot and $K_v = 40$, with a peak time of 0.1 second.

SOLUTION: The uncompensated system is $G(s) = 100K/[s(s+36)(s+100)]$. We will follow the outlined procedure.

1. We first look at the closed-loop bandwidth needed to meet the speed requirement imposed by $T_p = 0.1$ second. From Eq. (10.56), with $T_p = 0.1$ second and $\zeta = 0.456$ (i.e., 20% overshoot), a closed-loop bandwidth of 46.6 rad/s is required.
2. In order to meet the specification of $K_v = 40$, K must be set at 1440, yielding $G(s) = 144{,}000/[s(s+36)(s+100)]$.
3. The uncompensated system's frequency response plots for $K = 1440$ are shown in Figure 11.9.
4. A 20% overshoot implies a phase margin of 48.1°. The uncompensated system with $K = 1440$ has a phase margin of 34° at a phase-margin frequency

[4] We know that the phase-margin frequency will be increased after the insertion of the compensator. At this new phase-margin frequency, the system's phase will be smaller than originally estimated, as seen by comparing points B and D in Figure 11.7. Hence, an additional phase should be added to that provided by the lead compensator to correct for the phase reduction caused by the original system.

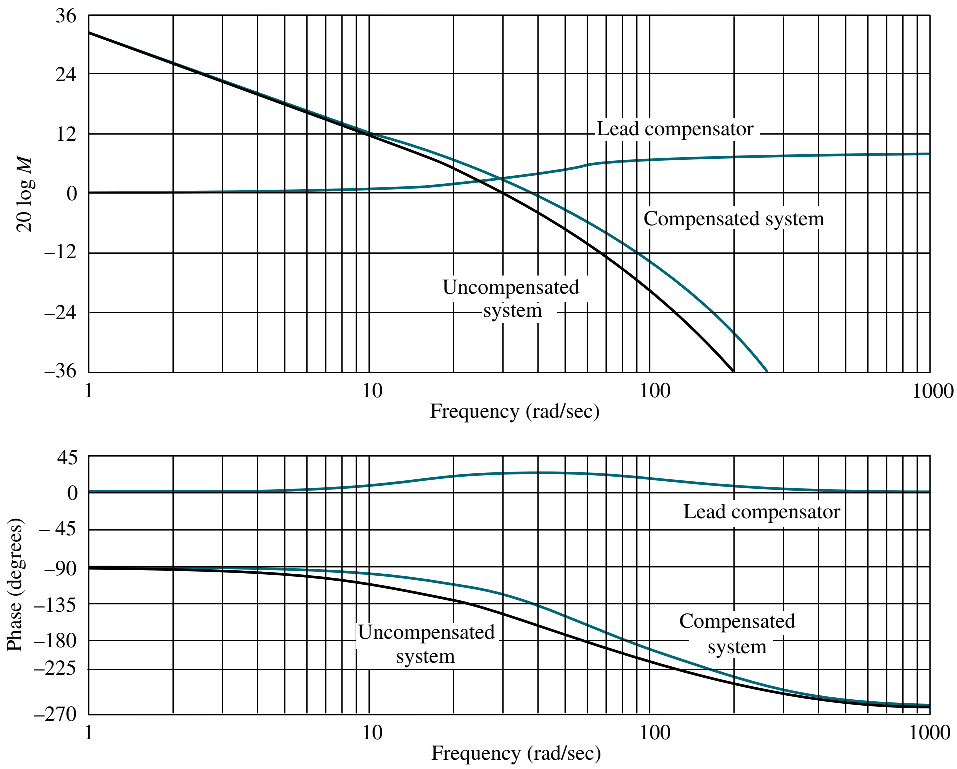


FIGURE 11.9 Bode plots for lead compensation in Example 11.3

of 29.6. To increase the phase margin, we insert a lead network that adds enough phase to yield a 48.1° phase margin. Since we know that the lead network will also increase the phase-margin frequency, we add a correction factor to compensate for the lower uncompensated system's phase angle at this higher phase-margin frequency. Since we do not know the higher phase-margin frequency, we assume a correction factor of 10°. Thus, the total phase contribution required from the compensator is $48.1° - 34° + 10° = 24.1°$. In summary, our compensated system should have a phase margin of 48.1° with a bandwidth of 46.6 rad/s. If the system's characteristics are not acceptable after the design, then a redesign with a different correction factor may be necessary.

5. Using Eq. (11.11), $\beta = 0.42$ for $\phi_{max} = 24.1°$.
6. From Eq. (11.12), the lead compensator's magnitude is 3.76 dB at ω_{max}.
7. If we select ω_{max} to be the new phase-margin frequency, the uncompensated system's magnitude at this frequency must be -3.76 dB to yield a 0 dB crossover at ω_{max} for the compensated system. The uncompensated system passes through -3.76 dB at $\omega_{max} = 39$ rad/s. This frequency is thus the new phase-margin frequency.
8. We now find the lead compensator's break frequencies. From Eq. (11.9), $1/T = 25.3$ and $1/\beta T = 60.2$.
9. Hence, the compensator is given by

$$G_c(s) = \frac{1}{\beta}\frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = 2.38\frac{s + 25.3}{s + 60.2} \tag{11.13}$$

where 2.38 is the gain required to keep the dc gain of the compensator at unity so that $K_v = 40$ after the compensator is inserted.

The final, compensated open-loop transfer function is then

$$G_c(s)G(s) = \frac{342{,}600(s+25.3)}{s(s+36)(s+100)(s+60.2)} \tag{11.14}$$

10. From Figure 11.9, the lead-compensated open-loop magnitude response is −7 dB at approximately 68.8 rad/s. Thus, we estimate the closed-loop bandwidth to be 68.8 rad/s. Since this bandwidth exceeds the requirement of 46.6 rad/s, we assume the peak time specification is met. This conclusion about the peak time is based upon a second-order and asymptotic approximation that will be checked via simulation.

11. Figure 11.9 summarizes the design and shows the effect of the compensation. Final results, obtained from a simulation and the actual (nonasymptotic) frequency response, are shown in Table 11.3. Notice the increase in phase margin, phase-margin frequency, and closed-loop bandwidth after the lead compensator was added to the gain-adjusted system. The peak time and the steady-state error requirements have been met, although the phase margin is less than that proposed and the percent overshoot is 2.6% larger than proposed. Finally, if the performance is not acceptable, a redesign is necessary.

TABLE 11.3 Characteristic of the lead-compensated system of Example 11.3

Parameter	Proposed specification	Actual gain-compensated value	Actual lead-compensated value
K_v	40	40	40
Phase margin	48.1°	34°	45.5°
Phase-margin frequency	—	29.6 rad/s	39 rad/s
Closed-loop bandwidth	46.6 rad/s	50 rad/s	68.8 rad/s
Percent overshoot	20	37	22.6
Peak time	0.1 second	0.1 second	0.075 second

MATLAB
ML

Students who are using MATLAB should now run ch11p3 in Appendix B. You will learn how to use MATLAB to design a lead compensator. You will enter the desired percent overshoot, peak time, and K_v. MATLAB then designs a lead compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.3 using MATLAB.

Skill-Assessment Exercise 11.3

WileyPLUS
WPCS
Control Solutions

PROBLEM: Design a lead compensator for the system in Skill-Assessment Exercise 11.1 to meet the following specifications: $\%OS = 20\%$, $T_s = 0.2$ s and $K_v = 50$.

ANSWER: $G_{\text{lead}}(s) = \dfrac{2.27(s+33.2)}{(s+75.4)}$; $G(s) = \dfrac{300{,}000}{s(s+50)(s+120)}$

The complete solution is at www.wiley.com/college/nise.

TryIt 11.3

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.3.

```
pos=20
Ts=0.2
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2))
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
Wbw=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2))
K=50*50*120
G=zpk([],[0,-50,-120],K)
sisotool(G,1)
```

When the **SISO Design for SISO Design Task Window** appears:

1. Right-click on the Bode plot area and select **Grid.**
2. Note the phase margin and bandwidth shown in the MATLAB **Command Window.**
3. On the **SISO Design for SISO Design Task Window toolbar,** click on the red pole.
4. Place the pole of the compensator by clicking on the gain plot at a frequency that is to the right of the desired bandwidth found in Step 2.
5. On the **SISO Design for SISO Design Task Window toolbar,** click on the red zero.
6. Place the zero of the compensator by clicking on the gain plot to the left of the desired bandwidth.
7. Reshape the Bode plots: alternately grab the pole and the zero with the mouse and alternately move them along the phase plot until the phase plot show a P.M. equal to that found in Step 2 and a phase-margin frequency close to the bandwidth found in Step 2.
8. Right-click in the Bode plot area and select **Edit Compensator** . . .
9. Read the lead compensator in the **Control and Estimation Tools Manager Window.**

Keep in mind that the previous examples were designs for third-order systems and must be simulated to ensure the desired transient results. In the next section, we look at lag-lead compensation to improve steady-state error and transient response.

11.5 Lag-Lead Compensation

In Section 9.4, using root locus, we designed lag-lead compensation to improve the transient response and steady-state error. Figure 11.10 is an example of a system to which lag-lead compensation can be applied. In this section we repeat the design, using frequency response techniques. One method is to design the lag compensation to lower the high-frequency gain, stabilize the system, and improve the steady-state error and then design a lead compensator to meet the phase-margin requirements. Let us look at another method.

Section 9.6 describes a passive lag-lead network that can be used in place of separate lag and lead networks. It may be more economical to use a single, passive network that performs both tasks, since the buffer amplifier that separates the lag network from the lead network may be eliminated. In this section, we emphasize lag-lead design, using a single, passive lag-lead network.

The transfer function of a single, passive lag-lead network is

$$G_c(s) = G_{\text{Lead}}(s)G_{\text{Lag}}(s) = \left(\frac{s+\dfrac{1}{T_1}}{s+\dfrac{\gamma}{T_1}}\right)\left(\frac{s+\dfrac{1}{T_2}}{s+\dfrac{1}{\gamma T_2}}\right) \tag{11.15}$$

(*a*) (*b*)

FIGURE 11.10 **a.** The National Advanced Driving Simulator at the University of Iowa; **b.** test driving the simulator with its realistic graphics (Katharina Bosse/laif/Redux Pictures.)

where $\gamma > 1$. The first term in parentheses produces the lead compensation, and the second term in parentheses produces the lag compensation. The constraint that we must follow here is that the single value γ replaces the quantity α for the lag network in Eq. (11.2) and the quantity β for the lead network in Eq. (11.6). For our design, α and β must be reciprocals of each other. An example of the frequency response of the passive lag-lead is shown in Figure 11.11.

We are now ready to enumerate a design procedure.

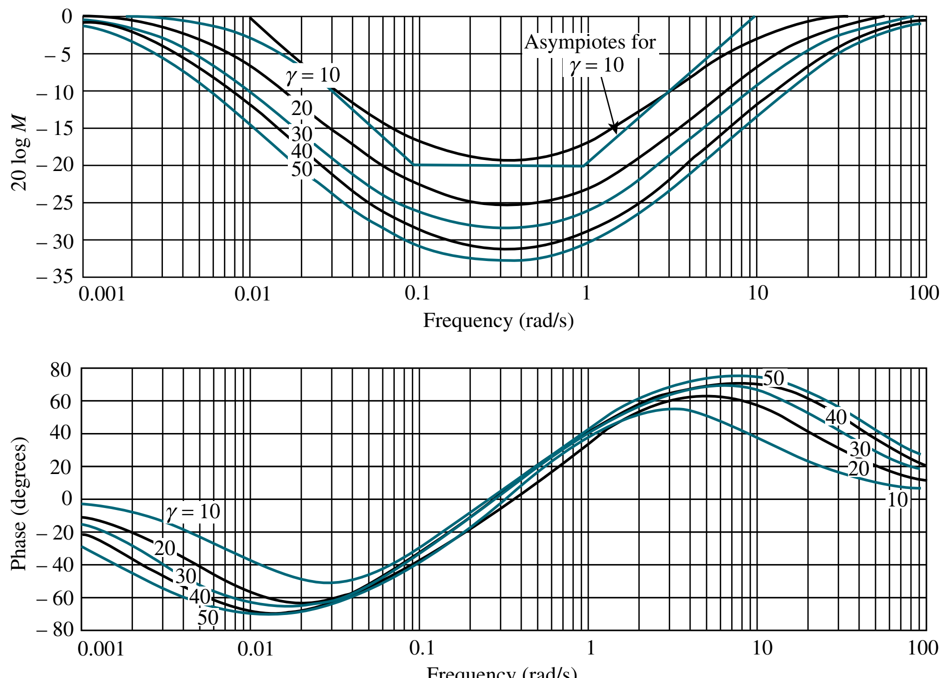


FIGURE 11.11 Sample frequency response curves for a lag-lead compensator, $G_c(s) = [(s+1)(s+0.1)]/\left[(s+\gamma)\left(s+\frac{0.1}{\gamma}\right)\right]$

Design Procedure

1. Using a second-order approximation, find the closed-loop bandwidth required to meet the settling time, peak time, or rise time requirement (see Eqs. (10.55) and (10.56)).
2. Set the gain, K, to the value required by the steady-state error specification.
3. Plot the Bode magnitude and phase diagrams for this value of gain.
4. Using a second-order approximation, calculate the phase margin to meet the damping ratio or percent overshoot requirement, using Eq. (10.73).
5. Select a new phase-margin frequency near ω_{BW}.
6. At the new phase-margin frequency, determine the additional amount of phase lead required to meet the phase-margin requirement. Add a small contribution that will be required after the addition of the lag compensator.
7. Design the lag compensator by selecting the higher break frequency one decade below the new phase-margin frequency. The design of the lag compensator is not critical, and any design for the proper phase margin will be relegated to the lead compensator. The lag compensator simply provides stabilization of the system with the gain required for the steady-state error specification. Find the value of γ from the lead compensator's requirements. Using the phase required from the lead compensator, the phase response curve of Figure 11.8 can be used to find the value of $\gamma = 1/\beta$. This value, along with the previously found lag's upper break frequency, allows us to find the lag's lower break frequency.
8. Design the lead compensator. Using the value of γ from the lag compensator design and the value assumed for the new phase-margin frequency, find the lower and upper break frequency for the lead compensator, using Eq. (11.9) and solving for T.
9. Check the bandwidth to be sure the speed requirement in Step 1 has been met.
10. Redesign if phase-margin or transient specifications are not met, as shown by analysis or simulation.

Let us demonstrate the procedure with an example.

Example 11.4

Lag-Lead Compensation Design

PROBLEM: Given a unity feedback system where $G(s) = K/[s(s+1)(s+4)]$, design a passive lag-lead compensator using Bode diagrams to yield a 13.25% overshoot, a peak time of 2 seconds, and $K_v = 12$.

SOLUTION: We will follow the steps previously mentioned in this section for lag-lead design.

1. The bandwidth required for a 2-seconds peak time is 2.29 rad/s.
2. In order to meet the steady-state error requirement, $K_v = 12$, the value of K is 48.
3. The Bode plots for the uncompensated system with $K = 48$ are shown in Figure 11.12. We can see that the system is unstable.
4. The required phase margin to yield a 13.25% overshoot is 55°.

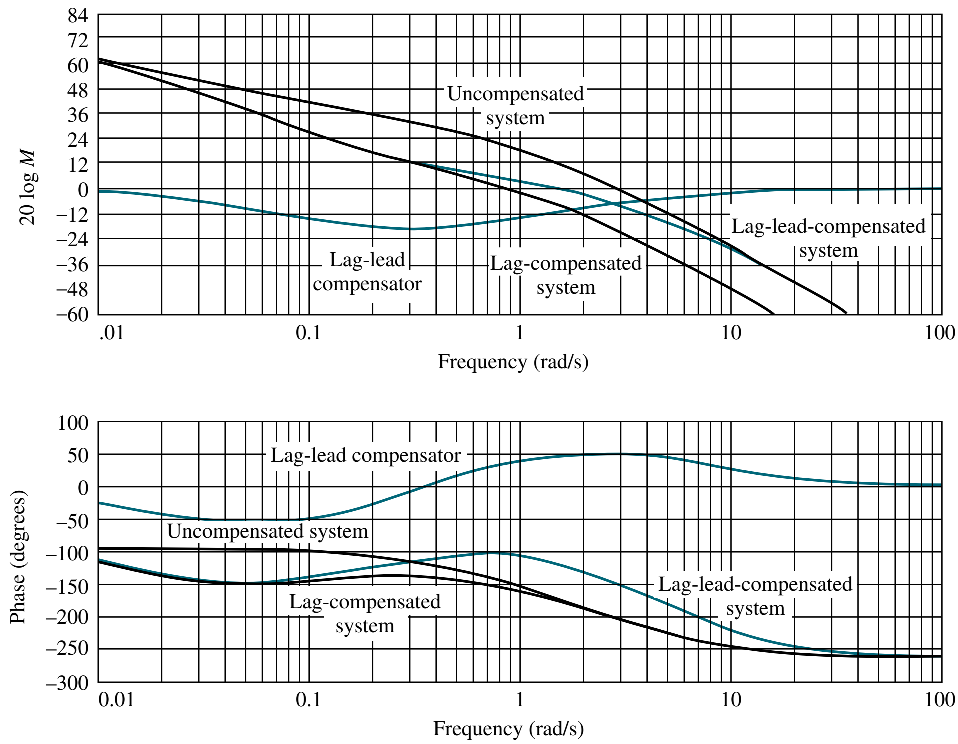


FIGURE 11.12 Bode plots for lag-lead compensation in Example 11.4

5. Let us select $\omega = 1.8$ rad/s as the new phase-margin frequency.
6. At this frequency, the uncompensated phase is $-176°$ and would require, if we add a $-5°$ contribution from the lag compensator, a $56°$ contribution from the lead portion of the compensator.
7. The design of the lag compensator is next. The lag compensator allows us to keep the gain of 48 required for $K_v = 12$ and not have to lower the gain to stabilize the system. As long as the lag compensator stabilizes the system, the design parameters are not critical since the phase margin will be designed with the lead compensator. Thus, choose the lag compensator so that its phase response will have minimal effect at the new phase-margin frequency. Let us choose the lag compensator's higher break frequency to be 1 decade below the new phase-margin frequency, at 0.18 rad/s. Since we need to add $56°$ of phase shift with the lead compensator at $\omega = 1.8$ rad/s, we estimate from Figure 11.8 that, if $\gamma = 10.6$ (since $\gamma = 1/\beta$, $\beta = 0.094$), we can obtain about $56°$ of phase shift from the lead compensator. Thus with $\gamma = 10.6$ and a new phase-margin frequency of $\omega = 1.8$ rad/s, the transfer function of the lag compensator is

$$G_{\text{lag}}(s) = \frac{1}{\gamma}\frac{\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{1}{\gamma T_2}\right)} = \frac{1}{10.6}\,\frac{(s+0.183)}{(s+0.0172)} \tag{11.16}$$

where the gain term, $1/\gamma$, keeps the dc gain of the lag compensator at 0 dB. The lag-compensated system's open-loop transfer function is

$$G_{\text{lag-comp}}(s) = \frac{4.53(s+0.183)}{s(s+1)(s+4)(s+0.0172)} \tag{11.17}$$

8. Now we design the lead compensator. At $\omega = 1.8$, the lag-compensated system has a phase angle of 180°. Using the values of $\omega_{\max} = 1.8$ and $\beta = 0.094$, Eq. (11.9) yields the lower break, $1/T_1 = 0.56$ rad/s. The higher break is then $1/\beta T_1 = 5.96$ rad/s. The lead compensator is

$$G_{\text{lead}}(s) = \gamma \frac{\left(s + \frac{1}{T_1}\right)}{\left(s + \frac{\gamma}{T_1}\right)} = 10.6\frac{(s+0.56)}{(s+5.96)} \tag{11.18}$$

The lag-lead-compensated system's open-loop transfer function is

$$G_{\text{lag-lead-comp}}(s) = \frac{48(s+0.183)(s+0.56)}{s(s+1)(s+4)(s+0.0172)(s+5.96)} \tag{11.19}$$

9. Now check the bandwidth. The closed-loop bandwidth is equal to that frequency where the open-loop magnitude response is approximately −7 dB. From Figure 11.12, the magnitude is −7 dB at approximately 3 rad/s. This bandwidth exceeds that required to meet the peak time requirement.

The design is now checked with a simulation to obtain actual performance values. Table 11.4 summarizes the system's characteristics. The peak time requirement is also met. Again, if the requirements were not met, a redesign would be necessary.

TABLE 11.4 Characteristics of gain-compensated system of Example 11.4

Parameter	Proposed specification	Actual value
K_v	12	12
Phase margin	55°	59.3°
Phase-margin frequency	—	1.63 rad/s
Closed-loop bandwidth	2.29 rad/s	3 rad/s
Percent overshoot	13.25	10.2
Peak time	2.0 seconds	1.61 seconds

MATLAB
ML

Students who are using MATLAB should now run ch11p4 in Appendix B. You will learn how to use MATLAB to design a lag-lead compensator. You will enter the desired percent overshoot, peak time, and K_v. MATLAB then designs a lag-lead compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.4 using MATLAB.

For a final example, we include the design of a lag-lead compensator using a Nichols chart. Recall from Chapter 10 that the Nichols chart contains a presentation of both the open-loop frequency response and the closed-loop frequency response. The axes of the Nichols chart are the open-loop magnitude and phase (y and x axis, respectively). The open-loop frequency response is plotted using the coordinates of the Nichols chart at each frequency. The open-loop plot is overlaying a grid that yields the closed-loop magnitude and phase. Thus, we have a presentation of both the open- and closed-loop responses. Thus, a design can be implemented that reshapes the Nichols plot to meet both open- and closed-loop frequency specifications.

From a Nichols chart, we can see simultaneously the following frequency response specifications that are used to design a desired time response: (1) phase margin, (2) gain margin, (3) closed-loop bandwidth, and (4) closed-loop peak amplitude.

In the following example, we first specify the following: (1) maximum allowable percent overshoot, (2) maximum allowable peak time, and (3) minimum allowable static error constant. We first design the lead compensator to meet the transient requirements followed by the lag compensator design to meet the steady-state error requirement. Although calculations could be made by hand, we will use MATLAB and SISOTOOL to make and shape the Nichols plot.

Let us first outline the steps that we will take in the example:

1. Calculate the damping ratio from the percent overshoot requirement using Eq. (4.39)
2. Calculate the peak amplitude, M_p, of the closed-loop response using Eq. (10.52) and the damping ratio found in (1).
3. Calculate the minimum closed-loop bandwidth to meet the peak time requirement using Eq. (10.56), with peak time and the damping ratio from (1).
4. Plot the open-loop response on the Nichols chart.
5. Raise the open-loop gain until the open-loop plot is tangent to the required closed-loop magnitude curve, yielding the proper M_p.
6. Place the lead zero at this point of tangency and the lead pole at a higher frequency. Zeros and poles are added in SISOTOOL by clicking either one on the tool bar and then clicking the position on the open-loop frequency response curve where you desire to add the zero or pole.
7. Adjust the positions of the lead zero and pole until the open-loop frequency response plot is tangent to the same M_p curve, but at the approximate frequency found in (3). This yields the proper closed-loop peak and proper bandwidth to yield the desired percent overshoot and peak time, respectively.
8. Evaluate the open-loop transfer function, which is the product of the plant and the lead compensator, and determine the static error constant.
9. If the static error constant is lower than required, a lag compensator must now be designed. Determine how much improvement in the static error constant is required.
10. Recalling that the lag pole is at a frequency below that of the lag zero, place a lag pole and zero at frequencies below the lead compensator and adjust to yield the desired improvement in static error constant. As an example, recall from Eq. (9.5) that the improvement in static error constant for a Type 1 system is equal to the ratio of the lag zero value divided by the lag pole value. Readjust the gain if necessary.

Example 11.5

MATLAB **ML**

Gui Tool **GUIT**

Lag-Lead Design Using the Nichols Chart, MATLAB, and SISOTOOL

PROBLEM: Design a lag-lead compensator for the plant, $G(s) = \dfrac{K}{s(s+5)(s+10)}$, to meet the following requirements: (1) a maximum of 20% overshoot, (2) a peak time of no more than 0.5 seconds, (3) a static error constant of no less than 6.

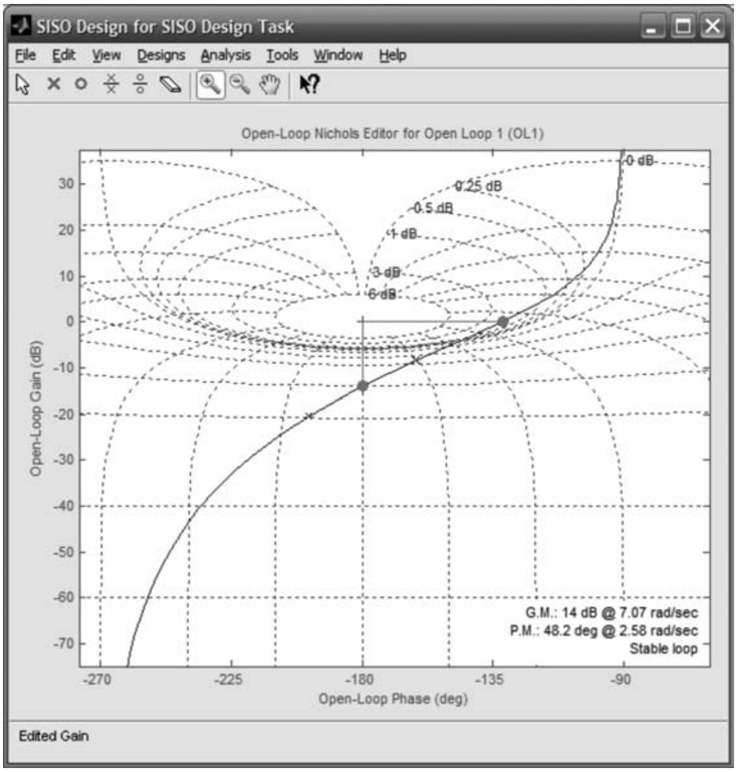


FIGURE 11.13 Nichols chart after gain adjustment

SOLUTION: We follow the steps enumerated immediately above,

1. Using Eq. (4.39), $\zeta = 0.456$ for 20% overshoot.
2. Using Eq. (10.52), $M_p = 1.23 = 1.81$ dB for $\zeta = 0.456$.
3. Using Eq. (10.56), $\omega_{BW} = 9.3$ r/s for $\zeta = 0.456$ and $T_p = 0.5$.
4. Plot the open-loop frequency response curve on the Nichols chart for $K = 1$.
5. Raise the open-loop frequency response curve until it is tangent to the closed-loop peak of 1.81 dB curve as shown in Figure 11.13. The frequency at the tangent point is approximately 3 r/s, which can be found by letting your mouse rest on the point of tangency. On the menu bar, select **Designs/Edit Compensator** . . . and find the gain added to the plant. Thus, the plant is now $G(s) = \dfrac{150}{s(s+5)(s+10)}$. The gain-adjusted closed-loop step response is shown in Figure 11.14. Notice that the peak time is about 1 second and must be decreased.
6. Place the lead zero at this point of tangency and the lead pole at a higher frequency.
7. Adjust the positions of the lead zero and pole until the open-loop frequency response plot is tangent to the same M_p curve, but at the approximate frequency found in **3**.
8. Checking **Designs/Edit Compensator** . . . shows $G(s)G_{\text{lead}}(s) = \dfrac{1286(s+1.4)}{s(s+5)(s+10)(s+12)}$, which yields a $K_v = 3$.

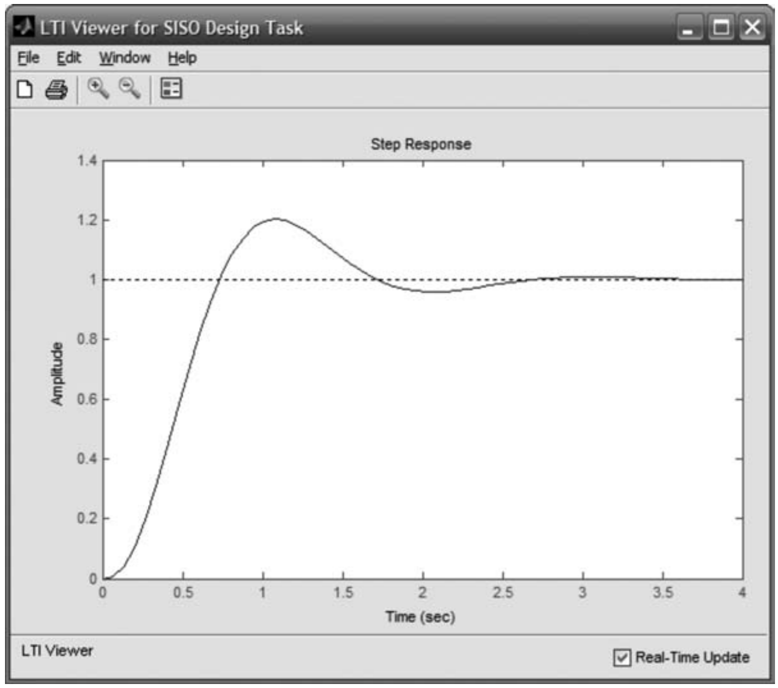


FIGURE 11.14 Gain-adjusted closed-loop step response

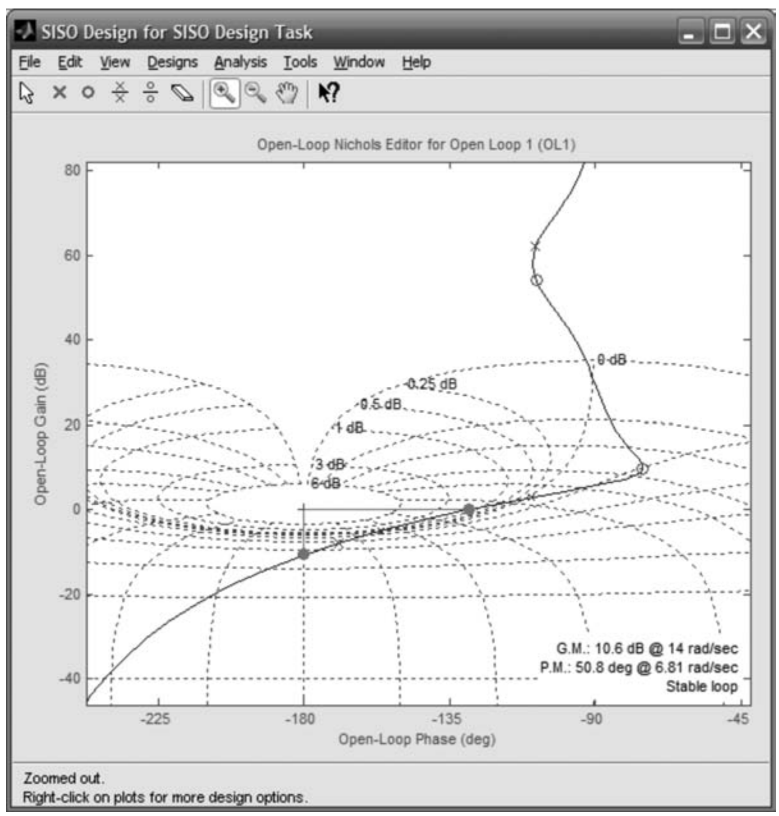


FIGURE 11.15 Nichols chart after lag-lead compensation

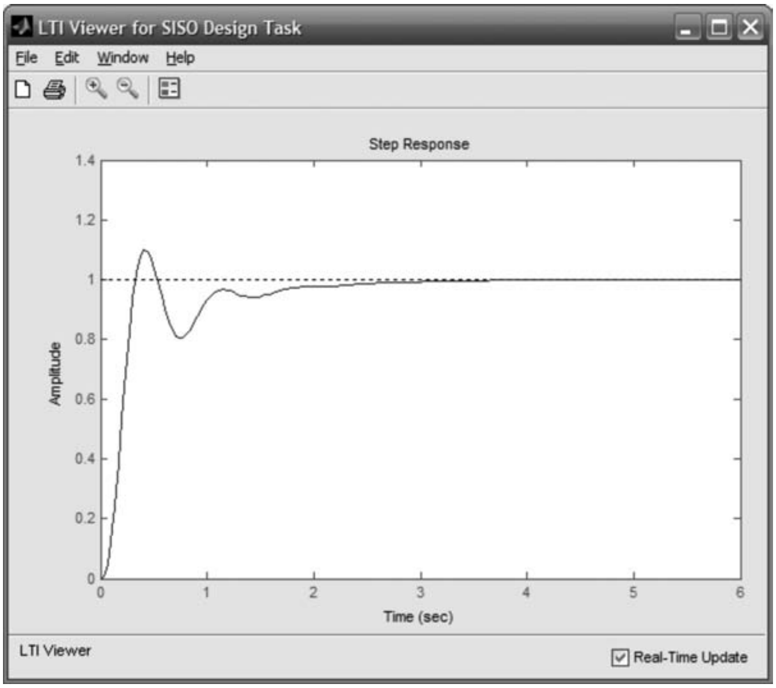


FIGURE 11.16 Lag-lead compensated closed-loop step response

9. We now add lag compensation to improve the static error constant by at least 2.
10. Now add a lag pole at −0.004 and a lag zero at −0.008. Readjust the gain to yield the same tangency as after the insertion of the lead. The final forward path is found to be $G(s)G_{\text{lead}}(s)G_{\text{lag}}(s) = \dfrac{1381(s+1.4)(s+0.008)}{s(s+5)(s+10)(s+12)(s+0.004)}$. The final Nichols chart is shown in Figure 11.15 and the compensated time response is shown in Figure 11.16. Notice that the time response has the expected slow climb to the final value that is typical of lag compensation. If your design requirements require a faster climb to the final response, then redesign the system with a larger bandwidth or attempt a design only with lead compensation. A problem at the end of the chapter provides the opportunity for practice.

Skill-Assessment Exercise 11.4

PROBLEM: Design a lag-lead compensator for a unity feedback system with the forward-path transfer function

$$G(s) = \frac{K}{s(s+8)(s+30)}$$

to meet the following specifications: $\%OS = 10\%$, $T_p = 0.6\,\text{s}$, and $K_v = 10$. Use frequency response techniques.

ANSWER: $G_{\text{lag}}(s) = 0.456\dfrac{(s+0.602)}{(s+0.275)}$; $G_{\text{lead}}(s) = 2.19\dfrac{(s+4.07)}{(s+8.93)}$; $K = 2400$.

The complete solution is at www.wiley.com/college/nise.

Case Studies

Our ongoing antenna azimuth position control system serves now as an example to summarize the major objectives of the chapter. The following cases demonstrate the use of frequency response methods to (1) design a value of gain to meet a percent overshoot requirement for the closed-loop step response and (2) design cascade compensation to meet both transient and steady-state error requirements.

Antenna Control: Gain Design

Design
D

PROBLEM: Given the antenna azimuth position control system shown on the front endpapers, Configuration 1, use frequency response techniques to do the following:

a. Find the preamplifier gain required for a closed-loop response of 20% overshoot for a step input.

b. Estimate the settling time.

SOLUTION: The block diagram for the control system is shown on the inside front cover (Configuration 1). The loop gain, after block diagram reduction, is

$$G(s) = \frac{6.63K}{s(s+1.71)(s+100)} = \frac{0.0388K}{s\left(\dfrac{s}{1.71}+1\right)\left(\dfrac{s}{100}+1\right)} \tag{11.20}$$

Letting $K = 1$, the magnitude and phase frequency response plots are shown in Figure 10.61.

a. To find K to yield a 20% overshoot, we first make a second-order approximation and assume that the second-order transient response equations relating percent overshoot, damping ratio, and phase margin are true for this system. Thus, a 20% overshoot implies a damping ratio of 0.456. Using Eq. (10.73), this damping ratio implies a phase margin of 48.1°. The phase angle should therefore be $(-180^\circ + 48.1^\circ) = -131.9^\circ$. The phase angle is -131.9° at $\omega = 1.49$ rad/s, where the gain is -34.1 dB. Thus $K = 34.1$ dB $= 50.7$ for a 20% overshoot. Since the system is third-order, the second-order approximation should be checked. A computer simulation shows a 20% overshoot for the step response.

b. Adjusting the magnitude plot of Figure 10.61 for $K = 50.7$, we find -7 dB at $\omega = 2.5$ rad/s, which yields a closed-loop bandwidth of 2.5 rad/s. Using Eq. (10.55) with $\zeta = 0.456$ and $\omega_{\text{BW}} = 2.5$, we find $T_s = 4.63$ seconds. A computer simulation shows a settling time of approximately 5 seconds.