Notice that since the pole–zero configuration is symmetrical about the real axis, the constant-gain loci are also symmetrical about the real axis.

Figure 6–29(b) shows the root loci and constant-gain loci for the system:

$$G(s) = \frac{K}{s(s+1)(s+2)}, \qquad H(s) = 1$$

Notice that since the configuration of the poles in the s plane is symmetrical about the real axis and the line parallel to the imaginary axis passing through point ($\sigma = -1$, $\omega = 0$), the constant-gain loci are symmetrical about the $\omega = 0$ line (real axis) and the $\sigma = -1$ line.

From Figures 6–29(a) and (b), notice that every point in the s plane has the corresponding K value. If we use a command rlocfind (presented next), MATLAB will give the K value of the specified point as well as the nearest closed-loop poles corresponding to this K value.

Finding the Gain Value *K* at an Arbitrary Point on the Root Loci. In MATLAB analysis of closed-loop systems, it is frequently desired to find the gain value K at an arbitrary point on the root locus. This can be accomplished by using the following rlocfind command:

```
[K, r] = rlocfind(num, den)
```

The rlocfind command, which must follow an rlocus command, overlays movable x-y coordinates on the screen. Using the mouse, we position the origin of the x-y coordinates over the desired point on the root locus and press the mouse button. Then MATLAB displays on the screen the coordinates of that point, the gain value at that point, and the closed-loop poles corresponding to this gain value.

If the selected point is not on the root locus, such as point A in Figure 6–29(a), the rlocfind command gives the coordinates of this selected point, the gain value of this point, such as $K = 2$, and the locations of the closed-loop poles, such as points B and C corresponding to this K value. [Note that every point on the s plane has a gain value. See, for example, Figures 6–29 (a) and (b).]

6–4 ROOT-LOCUS PLOTS OF POSITIVE FEEDBACK SYSTEMS

Root Loci for Positive-Feedback Systems.* In a complex control system, there may be a positive-feedback inner loop as shown in Figure 6–30. Such a loop is usually stabilized by the outer loop. In what follows, we shall be concerned only with the positive-feedback inner loop. The closed-loop transfer function of the inner loop is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

The characteristic equation is

$$1 - G(s)H(s) = 0 \qquad (6\text{–}17)$$

* Reference W-4

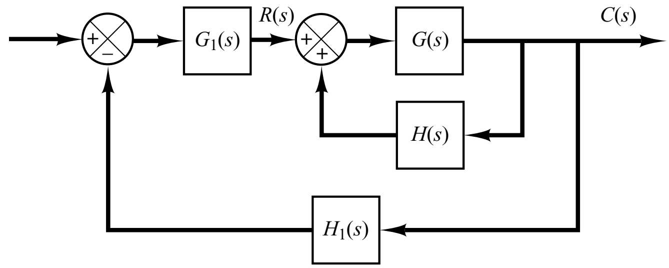


Figure 6–30
Control system.

This equation can be solved in a manner similar to the development of the root-locus method for negative-feedback systems presented in Section 6–2. The angle condition, however, must be altered.

Equation (6–17) can be rewritten as

$$G(s)H(s) = 1$$

which is equivalent to the following two equations:

$$\underline{/G(s)H(s)} = 0^\circ \pm k360^\circ \qquad (k = 0, 1, 2, \dots)$$

$$|G(s)H(s)| = 1$$

For the positive-feedback case, the total sum of all angles from the open-loop poles and zeros must be equal to $0^\circ \pm k360^\circ$. Thus the root locus follows a 0° locus in contrast to the 180° locus considered previously. The magnitude condition remains unaltered.

To illustrate the root-locus plot for the positive-feedback system, we shall use the following transfer functions $G(s)$ and $H(s)$ as an example.

$$G(s) = \frac{K(s+2)}{(s+3)(s^2+2s+2)}, \qquad H(s) = 1$$

The gain K is assumed to be positive.

The general rules for constructing root loci for negative-feedback systems given in Section 6–2 must be modified in the following way:

Rule 2 is Modified as Follows: If the total number of real poles and real zeros to the right of a test point on the real axis is even, then this test point lies on the root locus.

Rule 3 is Modified as Follows:

$$\text{Angles of asymptotes} = \frac{\pm k360^\circ}{n-m} \qquad (k = 0, 1, 2, \dots)$$

where n = number of finite poles of $G(s)H(s)$
m = number of finite zeros of $G(s)H(s)$

Rule 5 is Modified as Follows: When calculating the angle of departure (or angle of arrival) from a complex open-loop pole (or at a complex zero), subtract from 0° the sum of all angles of the vectors from all the other poles and zeros to the complex pole (or complex zero) in question, with appropriate signs included.

Other rules for constructing the root-locus plot remain the same. We shall now apply the modified rules to construct the root-locus plot.

1. Plot the open-loop poles ($s = -1 + j, s = -1 - j, s = -3$) and zero ($s = -2$) in the complex plane. As K is increased from 0 to ∞, the closed-loop poles start at the open-loop poles and terminate at the open-loop zeros (finite or infinite), just as in the case of negative-feedback systems.
2. Determine the root loci on the real axis. Root loci exist on the real axis between -2 and $+\infty$ and between -3 and $-\infty$.
3. Determine the asymptotes of the root loci. For the present system,

$$\text{Angles of asymptote} = \frac{\pm k360^\circ}{3 - 1} = \pm 180^\circ$$

   This simply means that asymptotes are on the real axis.
4. Determine the breakaway and break-in points. Since the characteristic equation is

$$(s + 3)(s^2 + 2s + 2) - K(s + 2) = 0$$

   we obtain

$$K = \frac{(s + 3)(s^2 + 2s + 2)}{s + 2}$$

   By differentiating K with respect to s, we obtain

$$\frac{dK}{ds} = \frac{2s^3 + 11s^2 + 20s + 10}{(s + 2)^2}$$

   Note that

$$\begin{aligned} 2s^3 + 11s^2 + 20s + 10 &= 2(s + 0.8)(s^2 + 4.7s + 6.24) \\ &= 2(s + 0.8)(s + 2.35 + j0.77)(s + 2.35 - j0.77) \end{aligned}$$

   Point $s = -0.8$ is on the root locus. Since this point lies between two zeros (a finite zero and an infinite zero), it is an actual break-in point. Points $s = -2.35 \pm j0.77$ do not satisfy the angle condition and, therefore, they are neither breakaway nor break-in points.
5. Find the angle of departure of the root locus from a complex pole. For the complex pole at $s = -1 + j$, the angle of departure θ is

$$\theta = 0^\circ - 27^\circ - 90^\circ + 45^\circ$$

   or

$$\theta = -72^\circ$$

   (The angle of departure from the complex pole at $s = -1 - j$ is 72°.)
6. Choose a test point in the broad neighborhood of the $j\omega$ axis and the origin and apply the angle condition. Locate a sufficient number of points that satisfy the angle condition.

Figure 6–31 shows the root loci for the given positive-feedback system. The root loci are shown with dashed lines and a curve.

Note that if

$$K > \left. \frac{(s + 3)(s^2 + 2s + 2)}{s + 2} \right|_{s=0} = 3$$

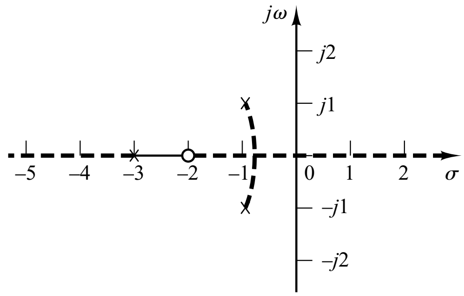


Figure 6–31
Root-locus plot for the positive-feedback system with $G(s) = K(s + 2)/[(s + 3)(s^2 + 2s + 2)]$, $H(s) = 1$.

one real root enters the right-half s plane. Hence, for values of K greater than 3, the system becomes unstable. (For $K > 3$, the system must be stabilized with an outer loop.)

Note that the closed-loop transfer function for the positive-feedback system is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$
$$= \frac{K(s + 2)}{(s + 3)(s^2 + 2s + 2) - K(s + 2)}$$

To compare this root-locus plot with that of the corresponding negative-feedback system, we show in Figure 6–32 the root loci for the negative-feedback system whose closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K(s + 2)}{(s + 3)(s^2 + 2s + 2) + K(s + 2)}$$

Table 6–2 shows various root-locus plots of negative-feedback and positive-feedback systems. The closed-loop transfer functions are given by

$$\frac{C}{R} = \frac{G}{1 + GH}, \qquad \text{for negative-feedback systems}$$
$$\frac{C}{R} = \frac{G}{1 - GH}, \qquad \text{for positive-feedback systems}$$

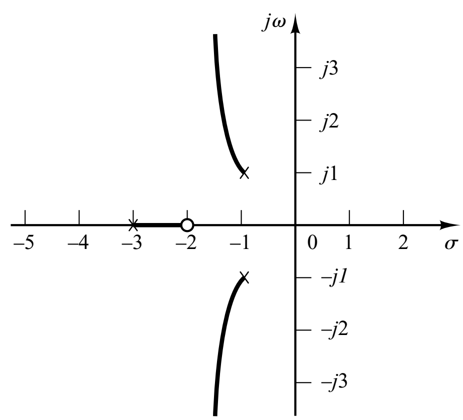
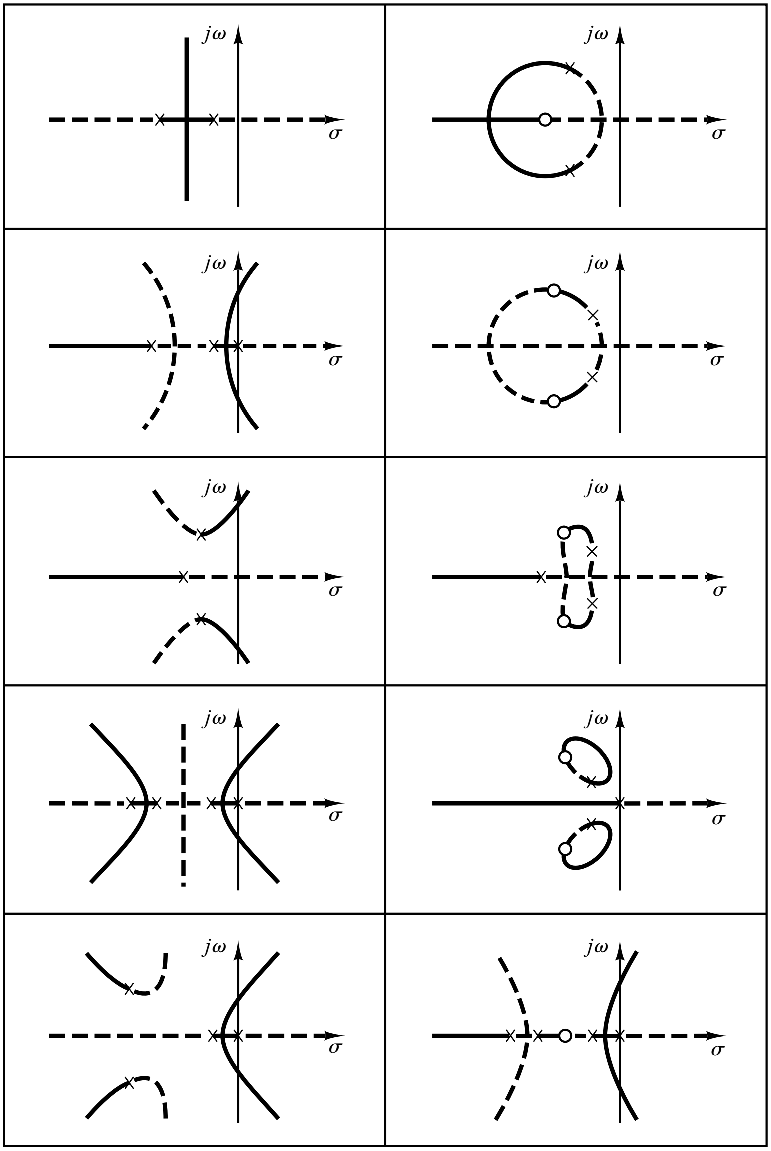


Figure 6–32
Root-locus plot for the negative-feedback system with $G(s) = K(s + 2)/[(s + 3)(s^2 + 2s + 2)]$, $H(s) = 1$.

where GH is the open-loop transfer function. In Table 6–2, the root loci for negative-feedback systems are drawn with heavy lines and curves, and those for positive-feedback systems are drawn with dashed lines and curves.

Table 6–2 Root-Locus Plots of Negative-Feedback and Positive-Feedback Systems



Heavy lines and curves correspond to negative-feedback systems; dashed lines and curves correspond to positive-feedback systems.

6–5 ROOT-LOCUS APPROACH TO CONTROL-SYSTEMS DESIGN

Preliminary Design Consideration. In building a control system, we know that proper modification of the plant dynamics may be a simple way to meet the performance specifications. This, however, may not be possible in many practical situations because the plant may be fixed and not modifiable. Then we must adjust parameters other than those in the fixed plant. In this book, we assume that the plant is given and unalterable.

In practice, the root-locus plot of a system may indicate that the desired performance cannot be achieved just by the adjustment of gain (or some other adjustable parameter). In fact, in some cases, the system may not be stable for all values of gain (or other adjustable parameter). Then it is necessary to reshape the root loci to meet the performance specifications.

The design problems, therefore, become those of improving system performance by insertion of a compensator. Compensation of a control system is reduced to the design of a filter whose characteristics tend to compensate for the undesirable and unalterable characteristics of the plant.

Design by Root-Locus Method. The design by the root-locus method is based on reshaping the root locus of the system by adding poles and zeros to the system's open-loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s plane. The characteristic of the root-locus design is its being based on the assumption that the closed-loop system has a pair of dominant closed-loop poles. This means that the effects of zeros and additional poles do not affect the response characteristics very much.

In designing a control system, if other than a gain adjustment (or other parameter adjustment) is required, we must modify the original root loci by inserting a suitable compensator. Once the effects on the root locus of the addition of poles and/or zeros are fully understood, we can readily determine the locations of the pole(s) and zero(s) of the compensator that will reshape the root locus as desired. In essence, in the design by the root-locus method, the root loci of the system are reshaped through the use of a compensator so that a pair of dominant closed-loop poles can be placed at the desired location.

Series Compensation and Parallel (or Feedback) Compensation. Figures 6–33(a) and (b) show compensation schemes commonly used for feedback control systems. Figure 6–33(a) shows the configuration where the compensator $G_c(s)$ is placed in series with the plant. This scheme is called *series compensation.*

An alternative to series compensation is to feed back the signal(s) from some element(s) and place a compensator in the resulting inner feedback path, as shown in Figure 6–33(b). Such compensation is called *parallel compensation* or *feedback compensation*.

In compensating control systems, we see that the problem usually boils down to a suitable design of a series or parallel compensator. The choice between series compensation and parallel compensation depends on the nature of the signals in the system, the power levels at various points, available components, the designer's experience, economic considerations, and so on.

In general, series compensation may be simpler than parallel compensation; however, series compensation frequently requires additional amplifiers to increase the gain and/or to provide isolation. (To avoid power dissipation, the series compensator is inserted at the lowest energy point in the feedforward path.) Note that, in general, the number of components required in parallel compensation will be less than the number of components

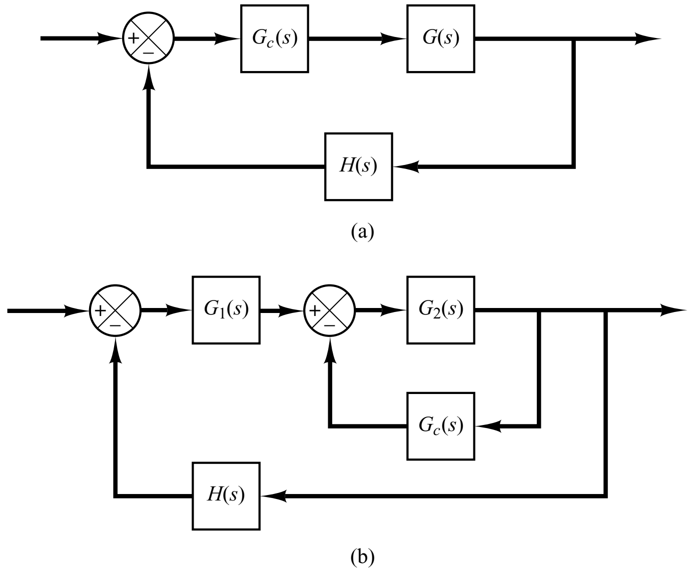


Figure 6–33
(a) Series compensation;
(b) parallel or feedback compensation.

in series compensation, provided a suitable signal is available, because the energy transfer is from a higher power level to a lower level. (This means that additional amplifiers may not be necessary.)

In Sections 6–6 through 6–9 we first discuss series compensation techniques and then present a parallel compensation technique using a design of a velocity-feedback control system.

Commonly Used Compensators. If a compensator is needed to meet the performance specifications, the designer must realize a physical device that has the prescribed transfer function of the compensator.

Numerous physical devices have been used for such purposes. In fact, many noble and useful ideas for physically constructing compensators may be found in the literature.

If a sinusoidal input is applied to the input of a network, and the steady-state output (which is also sinusoidal) has a phase lead, then the network is called a lead network. (The amount of phase lead angle is a function of the input frequency.) If the steady-state output has a phase lag, then the network is called a lag network. In a lag–lead network, both phase lag and phase lead occur in the output but in different frequency regions; phase lag occurs in the low-frequency region and phase lead occurs in the high-frequency region. A compensator having a characteristic of a lead network, lag network, or lag–lead network is called a lead compensator, lag compensator, or lag–lead compensator.

Among the many kinds of compensators, widely employed compensators are the lead compensators, lag compensators, lag–lead compensators, and velocity-feedback (tachometer) compensators. In this chapter we shall limit our discussions mostly to these types. Lead, lag, and lag–lead compensators may be electronic devices (such as circuits using operational amplifiers) or *RC* networks (electrical, mechanical, pneumatic, hydraulic, or combinations thereof) and amplifiers.

Frequently used series compensators in control systems are lead, lag, and lag–lead compensators. PID controllers which are frequently used in industrial control systems are discussed in Chapter 8.

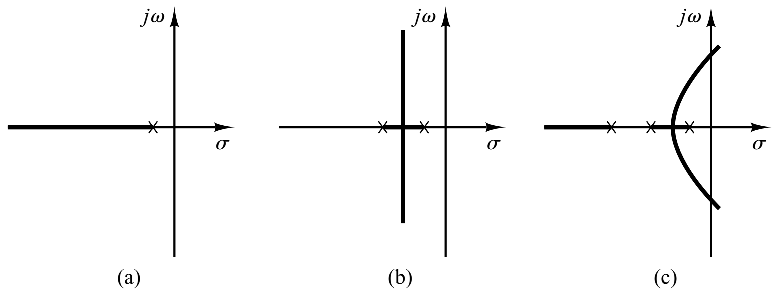


Figure 6–34
(a) Root-locus plot of a single-pole system; (b) root-locus plot of a two-pole system; (c) root-locus plot of a three-pole system.

It is noted that in designing control systems by the root-locus or frequency-response methods the final result is not unique, because the best or optimal solution may not be precisely defined if the time-domain specifications or frequency-domain specifications are given.

Effects of the Addition of Poles. The addition of a pole to the open-loop transfer function has the effect of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response. (Remember that the addition of integral control adds a pole at the origin, thus making the system less stable.) Figure 6–34 shows examples of root loci illustrating the effects of the addition of a pole to a single-pole system and the addition of two poles to a single-pole system.

Effects of the Addition of Zeros. The addition of a zero to the open-loop transfer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. (Physically, the addition of a zero in the feedforward transfer function means the addition of derivative control to the system. The effect of such control is to introduce a degree of anticipation into the system and speed up the transient response.) Figure 6–35(a) shows the root loci for a system

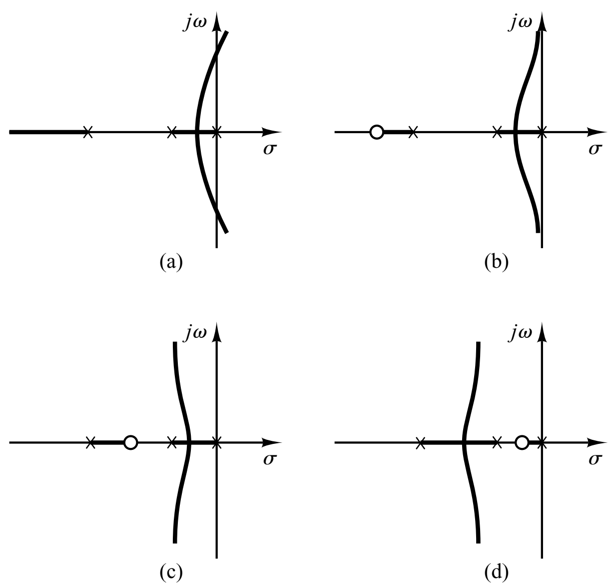


Figure 6–35
(a) Root-locus plot of a three-pole system; (b), (c), and (d) root-locus plots showing effects of addition of a zero to the three-pole system.