Lecture 6: Modelling – mechanical systems

- physical insight into the behaviour of mechanical systems is in general best obtained if the differential equations of motion are obtained by applying:

  1. Newtonian mechanics
  - define coordinates required to specify *configuration* of system
  - draw free body diagrams of system components showing all acting forces, including component interaction forces
  - write Newton's laws for each free body
  - write constitutive, compatibility and constraint equations
  - use compatibility equations to eliminate unwanted internal interaction force variables.

  2. Lagrangian mechanics
  - define generalised coordinates
  - define system boundary
  - write expressions for kinetic energy, potential energy and dissipation function
  - evaluate Lagrange's equation for each generalised coordinate.

6.1 Mechanical networks

- mechanical networks can be constructed using 'mechanical impedances'. The mechanical impedance is defined as the ratio of the applied force to the relative velocity over the element.

- for a mass element, Newton gives us $F(t) = m\ddot{x}$, thus

$$Z(s) = \frac{F(s)}{V(s)} = \frac{ms^2 X(s)}{sX(s)} = ms \qquad (6.1)$$

- for a spring element, we have $F(t) = kx$, thus

$$Z(s) = \frac{F(s)}{V(s)} = \frac{kX(s)}{sX(s)} = \frac{k}{s} \qquad (6.2)$$

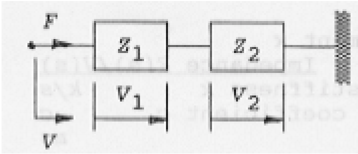
- for a damper element, we have $F(t) = c\dot{x}$, thus

$$Z(s) = \frac{F(s)}{V(s)} = \frac{csX(s)}{sX(s)} = c \qquad (6.3)$$

- given these basic definitions, we can now build up mechanical networks using simple rules that are analogous to those used in analysis electrical networks.

- when analysing mechanical networks the following combination rules apply:

1. <u>Series connection</u>



- using the relative velocities V_1 and V_2 across each element

$$F = Z_1 V_1 = Z_2 V_2 \tag{6.4}$$

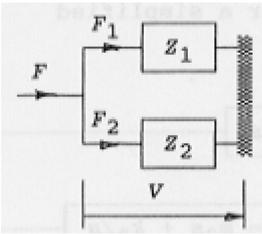
- hence $V_2 / V_1 = Z_1 / Z_2$ and

$$\begin{aligned} V &= V_1 + V_2 \\ &= V_1 \left(1 + V_2 / V_1\right) \\ &= V_1 \left(1 + Z_1 / Z_2\right) \end{aligned} \tag{6.5}$$

- thus, for mechanical impedances in series

$$\begin{aligned} Z = \frac{F}{V} &= \frac{Z_1 V_1}{1} \frac{1}{V_1 \left(1 + Z_1 / Z_2\right)} \\ &= \frac{Z_1 Z_2}{\left(1 + Z_1 / Z_2\right)} \end{aligned} \tag{6.6}$$

2. <u>Parallel connection</u>



- clearly

$$F = F_1 + F_2 \tag{6.7}$$

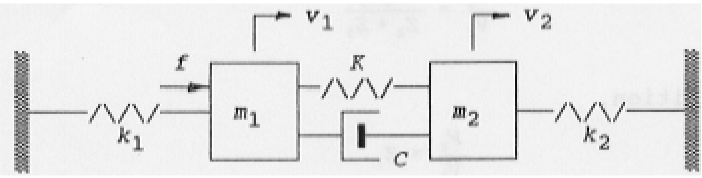
- where $F_1 = Z_1 V$ and $F_2 = Z_2 V$, giving

$$F = \left(Z_1 + Z_2\right) V \tag{6.8}$$

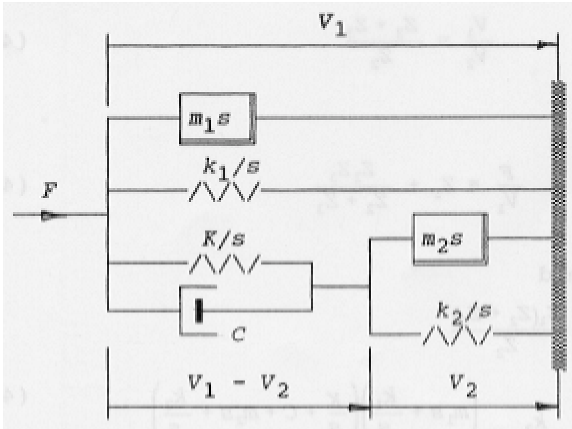
- thus, for mechanical impedances in parallel

$$Z = \frac{F}{V} = Z_1 + Z_2 \tag{6.9}$$

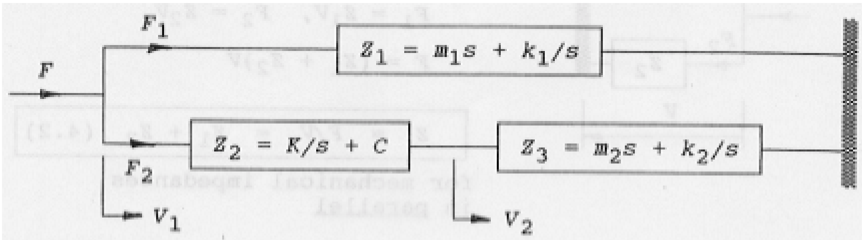
6.2 Example



- the equivalent mechanical network is



- to find the *transfer impedance* F/V_2, consider a simplified network diagram:



- from (6.6) we have

$$\frac{F_2}{V_1} = \frac{Z_2 Z_3}{Z_2 + Z_3} \tag{6.10}$$

- while, by definition

$$\frac{F_2}{V_2} = Z_3 \tag{6.11}$$

- so that

$$\frac{V_1}{V_2} = \frac{Z_2 + Z_3}{Z_2} \tag{6.12}$$

- whilst

$$\frac{F}{V_1} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \tag{6.13}$$

- equations (6.12) and (6.13) then yield

$$\frac{F}{V_2} = Z_3 + \frac{Z_1(Z_2 + Z_3)}{Z_2}$$

$$= m_2 s + \frac{k_2}{s} + \frac{\left(m_1 s + \frac{k_1}{s}\right)\left(\frac{K}{s} + C + m_2 s + \frac{k_2}{s}\right)}{\left(\frac{K}{s} + C\right)} \tag{6.14}$$

- the transfer function of interest may be the *receptance* $X_2(s)/F(s)$, where $X_2=V_2/s$. This follows from (6.14):

$$\begin{aligned}\frac{X_2}{F} &= \frac{V_2}{sF} \\ &= \frac{(Cs+K)}{(m_1s^2+k_1)(m_2s^2+Cs+k_2+K)+(m_2s^2+k_2)(Cs+K)}\end{aligned} \tag{6.15}$$