Lecture 3: Capacitors and Inductors

Capacitors and inductors do not dissipate but store energy, which can be retrieved later.

For this reason, capacitors and inductors are called storage elements.

3.1 Capacitors

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

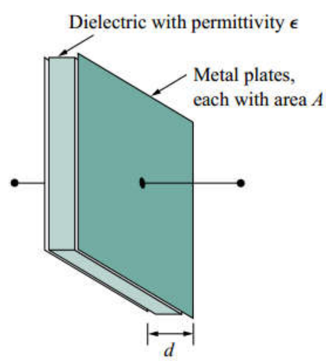

Figure 3.1

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

When a voltage source v is connected to the capacitor, as in Fig.3.2, the source deposits a positive charge q on one plate and a negative charge -q on the other. The capacitor is said to store the electric charge.

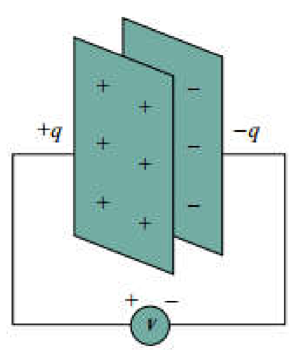

Figure 3.2

The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that,

$$q = Cv$$

Where C, the constant of proportionality, is known as the capacitance of the capacitor.

Thus**, Capacitance** is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Note from the equation above, 1 farad = 1 coulomb/volt.

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for
the parallel-plate capacitor shown in Fig. 6.1, the capacitance is given by

$$C = \frac{\epsilon A}{d}$$

Where A is the surface area of each plate, d is the distance between the plates, and is the permittivity of the dielectric material between the plates.

In general, three factors determine the value of the capacitance:
1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

Capacitors are commercially available in different values and types.
Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range.

They are described by the dielectric material they are made of and by whether they are of fixed or variable type.

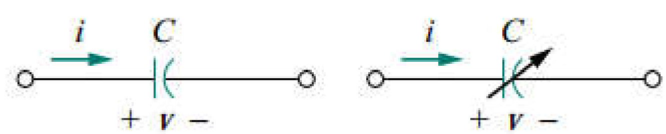

Figure 3.3: Circuit symbols for capacitors:
(a) fixed capacitor, (b) variable capacitor.

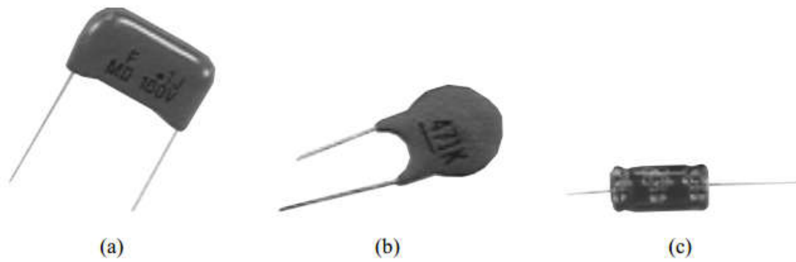

Figure 3.4: Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.

3.1.2: Energy Stored in a capacitor:

$$w = \frac{1}{2}Cv^2$$

$$w = \frac{q^2}{2C}$$

3.1.2: Properties of a capacitor

When the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus, a capacitor is an open circuit to dc

The voltage on the capacitor must be continuous. The voltage on a capacitor cannot change abruptly.

The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

A real, nonideal capacitor has a parallel-model leakage resistance.

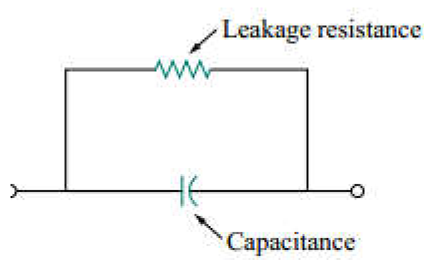

Figure 3.5: Circuit model of a nonideal capacitor.

Example 2.1

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

$$(a) Since\ q = Cv,$$

$$q = 3 \times 10^{-1\ 2} \times 20 = 60pC$$

$$(b) The\ energy\ stored\ is;$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-1\ 2} \times 400 = 600pJ$$

Exercise 3.1

What is the voltage across a 3-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored?

3.2: Inductor

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.
Any conductor of electric current has inductive properties and may be regarded as an inductor.

But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire

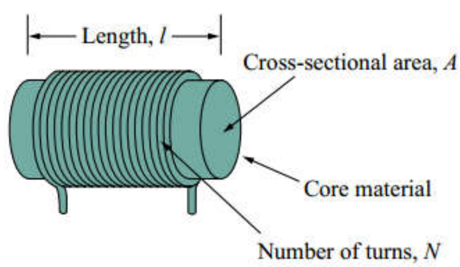

Figure 3.6: Typical form of an inductor.

An inductor consists of a coil of conducting wire.

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

$$v = L\frac{di}{dt}$$

Where L is the constant of proportionality called the inductance of the inductor.

The unit of inductance is the henry (H)

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H)

The inductance of an inductor depends on its physical dimension and construction.

For example, for the inductor (solenoid) shown in figure 3.6.

$$L = \frac{N^2\mu}{\ell}$$

Where N is the number of turns, L is the length, A is the cross-sectional area, and μ is the permeability of the core.

We can see from the equation above that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil.

Like capacitors, commercially available inductors come in different values and types. Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems. Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms coil and choke are also used for inductors.

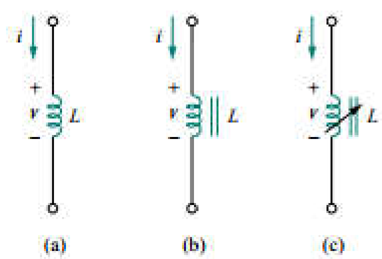

Figure 3.7: Circuit symbols for inductors:

(a) air-core, (b) iron-core, (c) variable iron-core

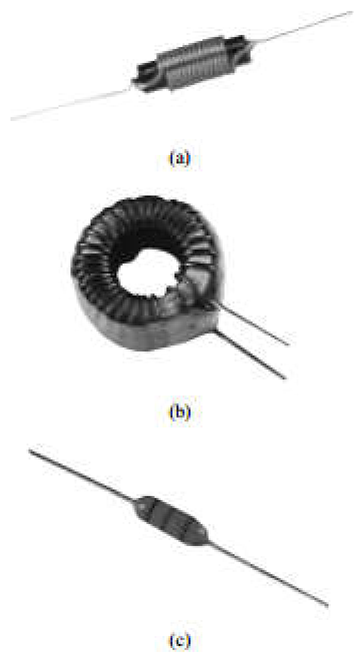

Figure 3.8: Various types of inductors:
(a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor

3.2.1 Energy stored in an inductor:

$$w = \frac{1}{2} L i^2$$

Where L is inductance, i is the current.

Properties of an inductor

The voltage across an inductor is zero when the current is constant thus an inductor acts like a short circuit to dc.

An important property of the inductor is its opposition to the change in current flowing through it. The current through an inductor cannot change instantaneously.

Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved later.
The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

A practical, nonideal inductor has a significant resistive component, as shown in Fig. 6.26. This is because the inductor is made of a conducting material such as copper, which has some resistance.

3.3: Applications

Capacitors and inductors possess the following three special properties that make them very useful in electric circuits:

1. The capacity to store energy makes them useful as temporary voltage or current sources. Thus, they can be used for generating a large amount of current or voltage for a short period.

2. Capacitors oppose any abrupt change in voltage; while inductors oppose any abrupt change in current. This property makes inductors useful for spark or arc suppression and for converting pulsating dc voltage into relatively smooth dc voltage.

3. Capacitors and inductors are frequency sensitive. This property makes them useful for frequency discrimination.