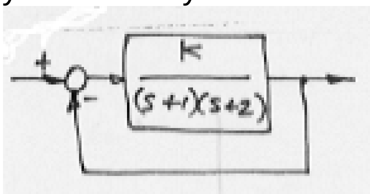
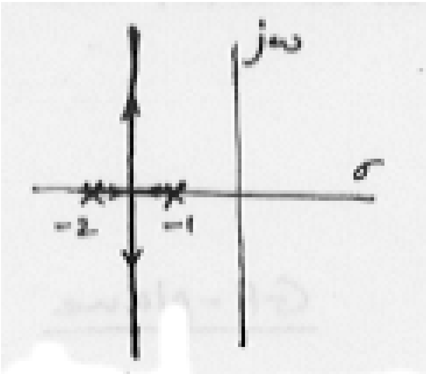
Lecture 20: stability analysis using Nyquist plots

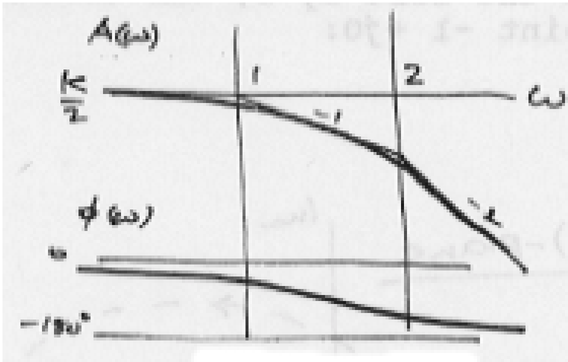
Example 1 – open and closed loop stable system

- consider the following unity feedback system:

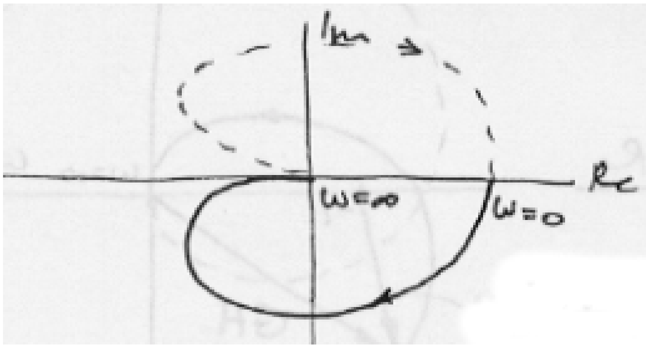


- which has the following root locus and open loop Bode diagrams:





- using the Bode diagram, we can then sketch the Nyquist plot:



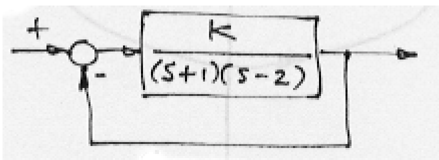
- from which it can be deduced that the system is closed loop stable for all values of K because there are no encirclements of the point $-1+i0$ ie. there are never any closed loop pole (zeros of $A(s)$) in the RHP:

$$N = 0,\ P = 0 \Rightarrow Z = 0$$

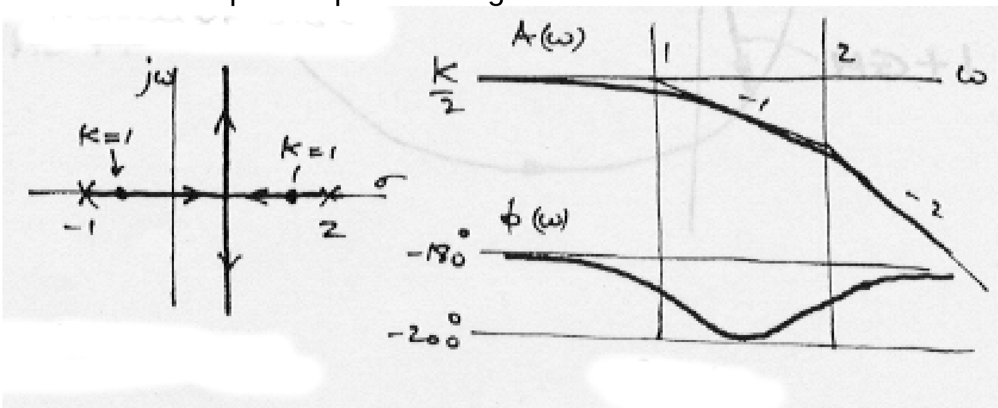
- this agrees with the root locus.

Example 2 - open and closed loop unstable system

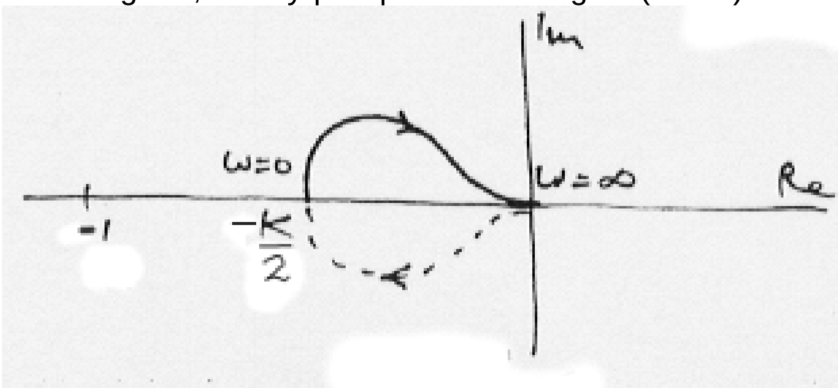
- consider a similar unity feedback system which is now open loop unstable:



- the root locus and open loop Bode diagrams are then:



- from the Bode diagram, the Nyquist plot for small gain ($K < 2$) is:



- from which it can be deduced that the system is closed loop unstable for *all* values of K:

  1. for $K < 2$: $N = 0,\ P = 1 \Rightarrow Z = 1$
  2. for $K > 2$: $N = 1,\ P = 1 \Rightarrow Z = 2$

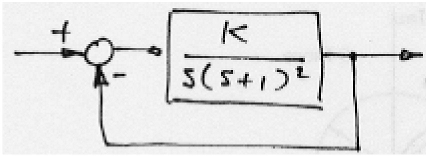
- which also agrees with the root locus.

Example 3 – system with singularities on the imaginary axis

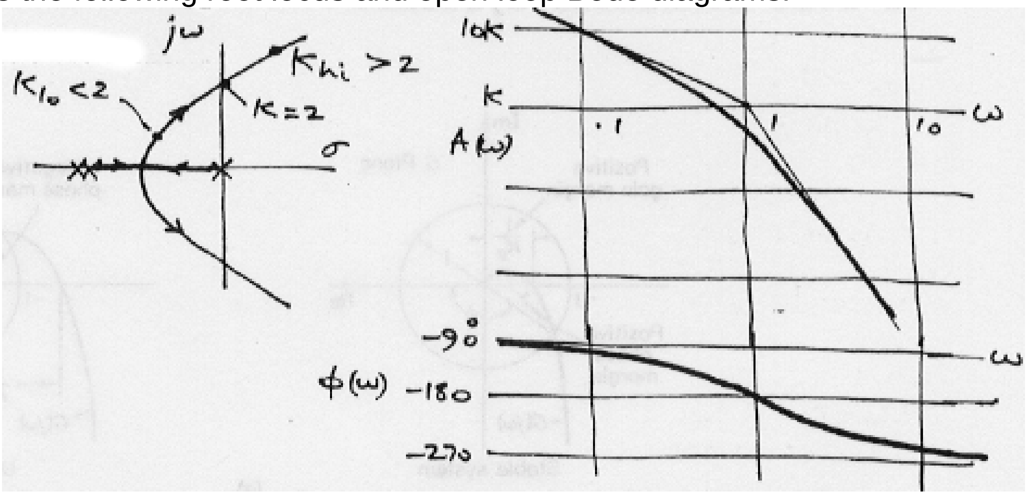
- if there are open-loop poles or zeros on the $i\omega$ axis, the evaluation contour must not pass through them. This is achieved by detouring from the $i\omega$ axis along infinitesimally small semi-circles into the RHP:

  1. the semicircles can be made small enough so as not to exclude any singularities of $A(s)$ in the RHP.
  2. as s moves along the semicircle, the phase of $GH(s)$ due to the detoured pole or zero will change by 180°.
  3. if the detoured pole is at the origin, it will cause the magnitude of $GH(i\omega)$ to become infinite, and the 180° phase change will correspond to an infinite semicircle in the $GH(i\omega)$ plane.

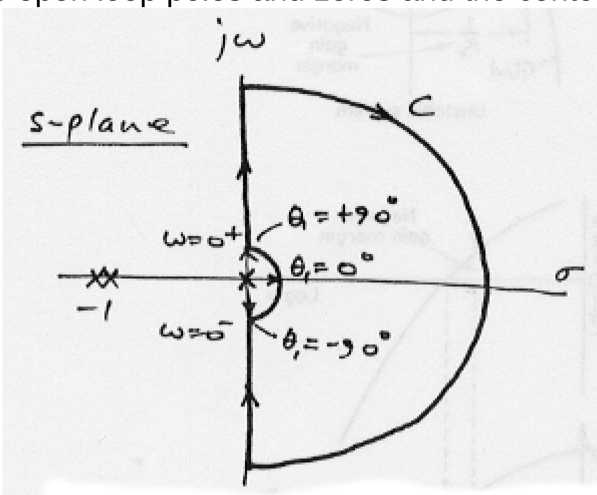
- for example, the system:



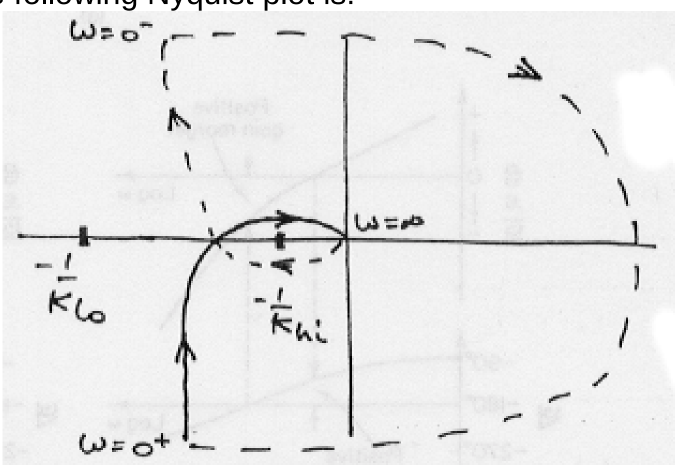
- has the following root locus and open loop Bode diagrams:



- in the s plane, the open loop poles and zeros and the contour are:



- which gives the following Nyquist plot is:



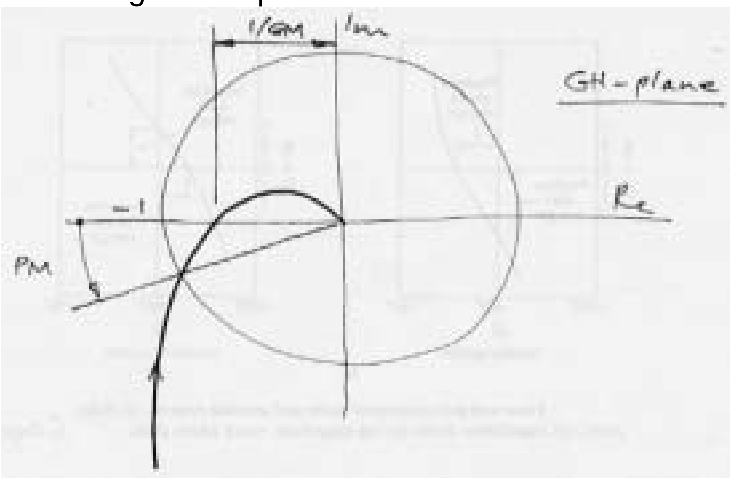
- thus, we have a system that is stable at lower gains and unstable at higher gains:

  1. for low gain $(K < 2)$: $N = 0,\ P = 0 \Rightarrow Z = 0$
  2. for high gain $(K > 2)$: $N = 0,\ P = 2 \Rightarrow Z = 2$

- which once again agrees with the root locus.

20.1 Relationship to the gain and phase margins

- the gain and phase margins are essentially measures of how close the Nyquist plot comes to encircling the -1 point:

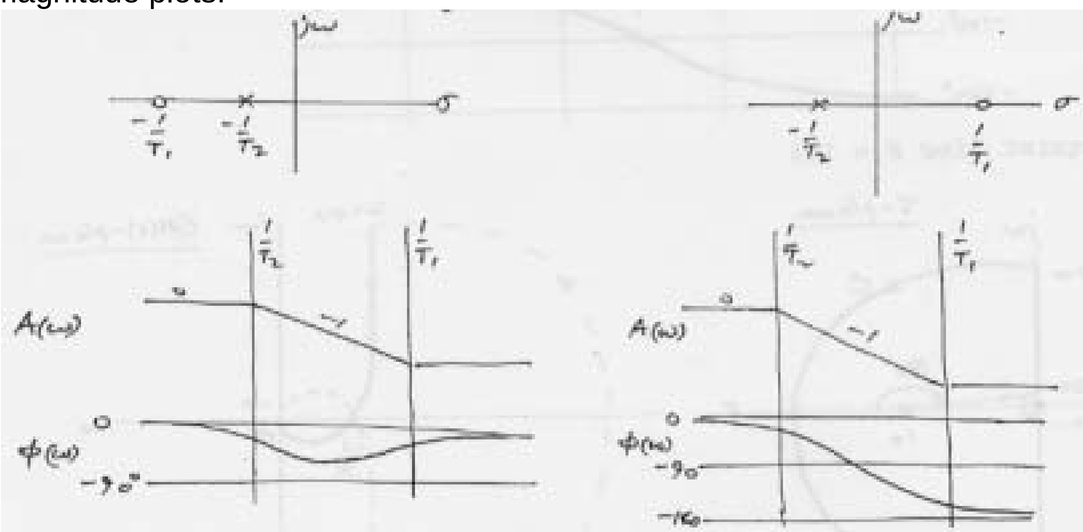


20.2 Non-minimum phase systems

- a system whose OLTF has no poles or zeros in the RHP is called a minimum phase system; one with open-loop poles or zeros in the RHP is called non-minimum phase. The terminology comes from the behaviour of the open-loop frequency response function: the range in phase angle of a minimum phase system is the smallest of any system with the same magnitude characteristics.

Example

- compare the phase plots for the following OLTFs which have the same magnitude plots:



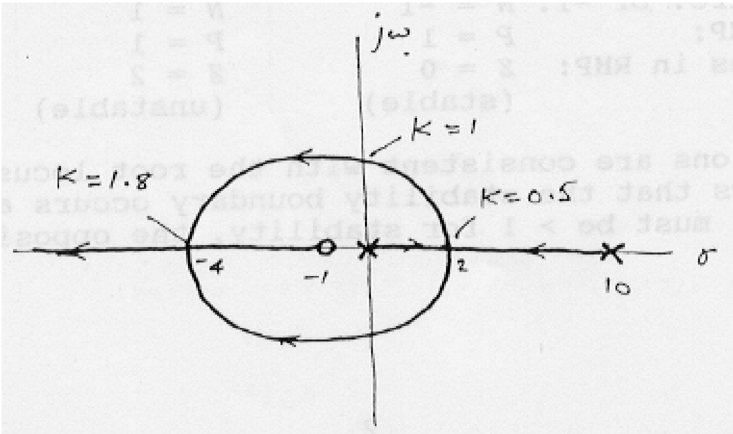
- applying the stability criteria for Root Locus and Nyquist plots is quite straightforward, but the interpretation of the Bode plots for non-minimum phase systems requires some care.

Example

- for example, consider a system with

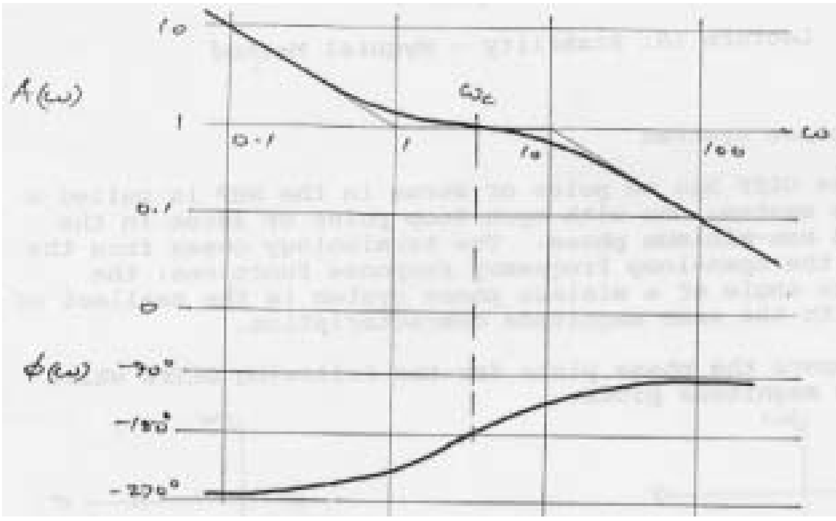
$$GH(s) = K\frac{10(s+1)}{s(s-10)}$$

- with the root locus:

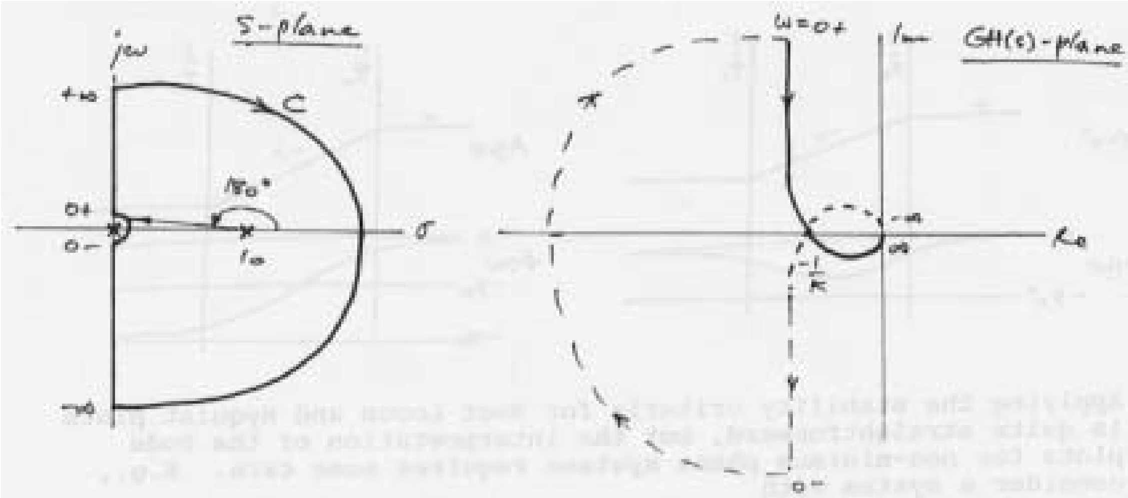


- and open loop Bode plots (in this case $K = 1$):

$$GH = K\cdot\frac{10(s+1)}{s(s-10)}$$
$$= K\frac{(s+1)}{s(s/10-1)}$$



- thus, the Nyquist plot for $K = 1$ is :



- note that for $K = 1$ the system is neutrally stable. The Nyquist plot also yields:

  1. for high gain $(K > 1)$: $N = -1,\ P = 1 \Rightarrow Z = 0 \Rightarrow \mathit{stable}$
  2. for low gain $(K < 1)$: $N = 1,\ P = 1 \Rightarrow Z = 0 \Rightarrow \mathit{unstable}$

- these conclusions are consistent with the root locus. The Bode plot also shows that the stability boundary occurs at $K = 1$, but that $|GH(i\omega_1)|$ must be >1 for stability, the opposite of the normal case.