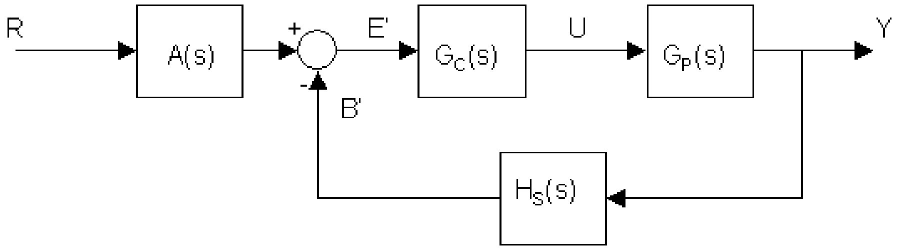
Lecture 14: The effects of feedback-Steady State Erros.

- consider the block diagram of a typical single-loop system:



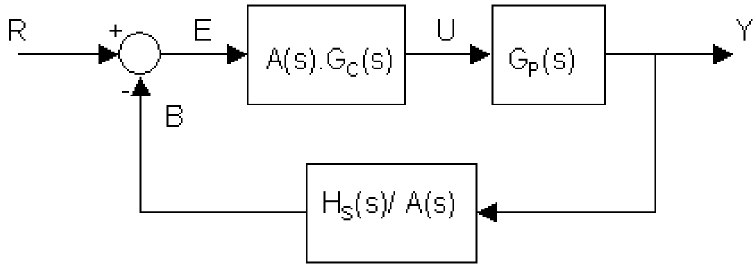
- where each block represents:

$A(s)$	input elements, prefiltering
$G_C(s)$	controller and actuator dynamics
$G_P(s)$	plant dynamics
$H_S(s)$	sensor dynamics, feedback elements of controller

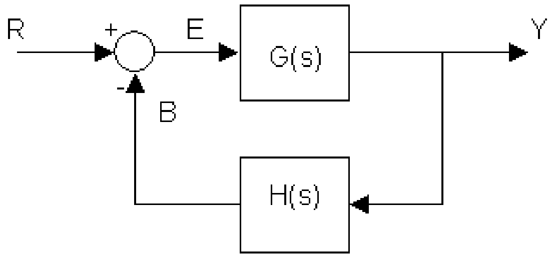
- and each signal is:

R	reference (command) input
B'	sensor output
E'	actuating error
U	plant input
Y	controlled variable
Y_E	system error

- this single loop system can be simplified by block diagram reduction:



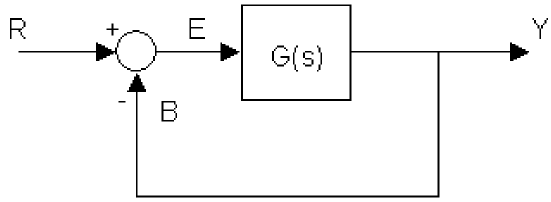
- where $B = B'/A$. Simplifying again:



- where, in terms of the original system blocks:

$G(s) = A(s)G_C(s)G_P(s)$	forward path transfer function
$H(s) = H_S(s)/A(s)$	feedback path transfer function

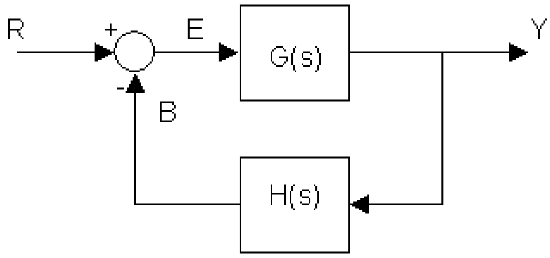
- note that if $A(s) = H_S(s)$, to within the accuracy of analysis, we have a <u>unity feedback system:</u>



- for which system error $Y_E = R - Y$ is the same as actuating error $E = R - B$

14.1 Closed-loop response

- Using:



- we see that

$$\begin{aligned} Y &= G.E \\ &= G(R - B) \\ &= G(R - Y.H) \\ \Rightarrow Y(1 + G.H) &= G.R \end{aligned} \tag{14.1}$$

- thus, the <u>closed loop transfer function</u> (CLTF) is

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}, \tag{14.2}$$

- the <u>open loop transfer function</u> (OLTF) is

$$\frac{B(s)}{E(s)} = G(s)H(s), \tag{14.3}$$

- and the <u>closed loop characteristic polynomial</u> is

$$\Delta(s) = 1 + G(s)H(s) \tag{14.4}$$

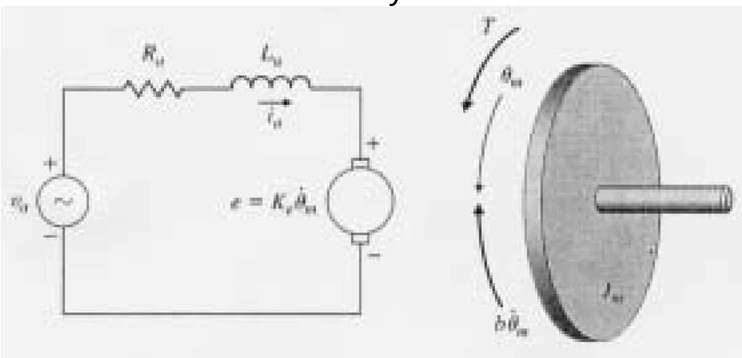
- other closed-loop transfer functions are:

$$\frac{E}{R} = \frac{1}{1 + GH}; \ \frac{Y_E}{R} = \frac{Y - R}{R} = \frac{1 + G(H - 1)}{1 + GH}; \ \frac{B}{R} = \frac{GH}{1 + GH} \tag{14.5}$$

- note the important feature that, as equations (14.2) and (14.5) show, <u>all</u> closed-loop TFs have the same characteristic polynomial $\Delta(s)$

14.2 Effects of feedback

- consider the mechanical and electrical dynamics of a DC motor:



- where:

v_a	applied voltage
R_a	armature resistance
L_a	armature inductance
$e = K_e\Omega_m$	back EMF of motor
$T = K_i i_a$	torque applied to motor
Ω_m	angular velocity of rotor
J_m	rotor moment of inertia
b	viscous friction coefficient

- from Newton, the rotor dynamics are governed by

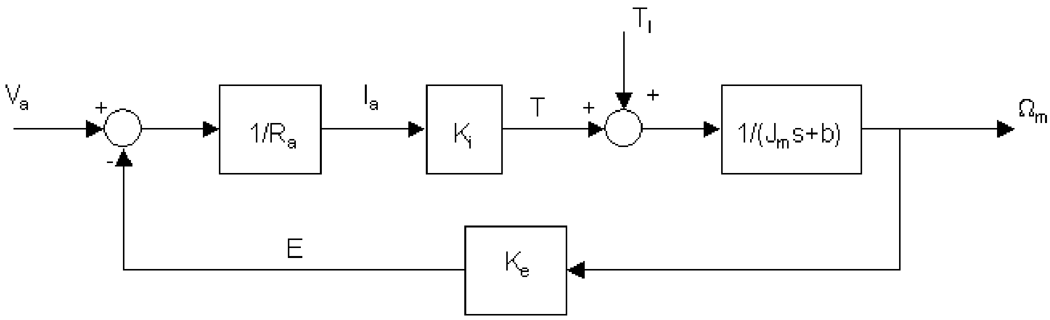
$$J_m\dot{\Omega}_m + b\Omega_m = K_i i_a + T_l \tag{14.6}$$

- where T_l is a load torque disturbance

- the armature inductance is often small, and is then neglected. Using KVL, we get

$$K_e\Omega_m + R_a i_a = v_a \tag{14.7}$$

- a block diagram of equations (14.6) and (14.7) is



- Laplace transforming equations (14.6) and (14.7) gives

$$\begin{aligned}(sJ_m + b)\Omega_m &= K_i I_a + T_l \\ K_e\Omega_m + R_a I_a &= V_a\end{aligned} \tag{14.8}$$

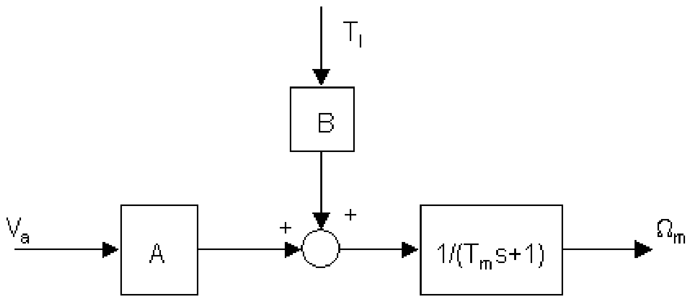
- solving for I_a and expressing equations (14.8) as a single equation gives

$$\Omega_m = \frac{A}{(T_m s+1)} V_a \qquad \text{command response}$$
$$+\frac{B}{(T_m s+1)} T_l \qquad \text{disturbance response} \tag{14.9}$$

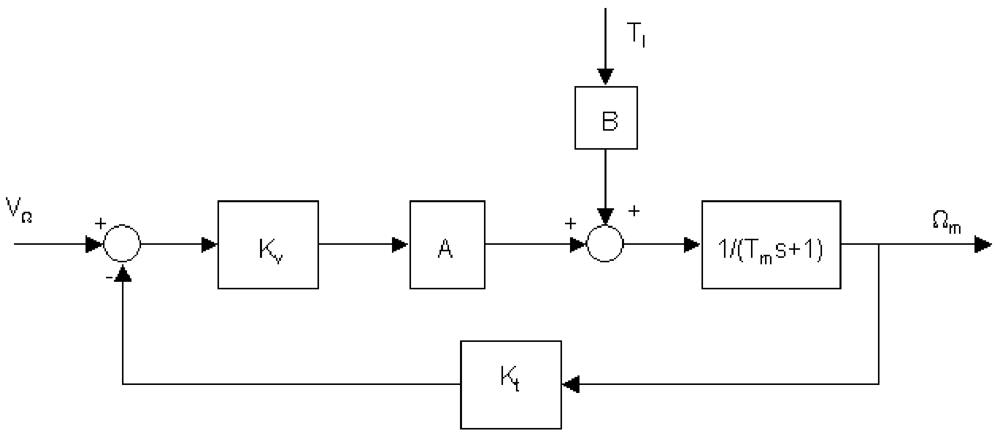
- where

$$A = \frac{K_i}{(bR_a + K_i K_e)}$$
$$B = \frac{R_a}{(bR_a + K_i K_e)} \tag{14.10}$$
$$T_m = \frac{J_m R_a}{(bR_a + K_i K_e)}$$

- thus, the block diagram above can be simplified to



- velocity (ie. tachometric) feedback is often introduced by using a small permanent magnet machine that produces a voltage that is proportional to the rotor speed. The block diagram then becomes



- where K_v represents the ‘velocity error amplifier’. The closed loop transfer functions are

$$\Omega_m = \frac{A'}{(T_m' s+1)} V_\Omega + \frac{B'}{(T_m' s+1)} T_l \tag{14.11}$$

- where

$$A' = \frac{K_v A}{(K_v A K_t + 1)}$$

$$B' = \frac{B}{(K_v A K_t + 1)} \qquad (14.12)$$

$$T_m^{'} = \frac{T_m}{(K_v A K_t + 1)}$$

- equations (14.9) and (14.11) contain only first order lags, and therefore represent purely exponential time responses.

14.2.1 Steady state command response

- consider now the steady state command response ($T_l = 0$):

$$\left(\frac{\Omega_m}{V_\Omega}\right)_{SS} = A' = \frac{K_v A}{(K_v A K_t + 1)} \qquad (14.13)$$

- if $K_v A K_t \gg 1$, which is deliberately achieved by setting $K_v \gg 1$, equation (14.13) becomes

$$\left(\frac{\Omega_m}{V_\Omega}\right)_{SS} = \frac{1}{K_t} \qquad (14.14)$$

- ie. the steady state response of the motor becomes insensitive to variations in any term that describes the motor itself. This may be a very desirable feature since motor wear, for example, becomes unimportant in terms of setting the steady state speed.

- note the importance of high quality sensing – if the tachometer degrades (ie. K_t changes), then we are really in trouble!

- *thus, two illustrated important characteristics of feedback are that it:*

  1. *changes the overall gain*

  2. *reduces the sensitivity to parameter variations*

14.2.2 Transient command response

- equation (14.11) gives the transient command response

$$\frac{\Omega_m}{V_\Omega} = \frac{A'}{(T_m^{'} s + 1)} \qquad (14.15)$$

- where from equation (14.12), $T_m^{'} = \frac{T_m}{(K_v A K_t + 1)}$. Thus, $T_m^{'} \to 0$ as $K_v A K_t \to \infty$

- consider now the unit step response of equation (14.15):

$$\omega_m(t) = A'\left[1 - \exp\left(-t/T_m^{'}\right)\right] \tag{14.16}$$

- ie. as $T_m^{'} \to 0$, the pole in equation (14.15) becomes 'faster', and the motor reaches its steady state speed more quickly.

- *thus, another important effect of feedback is that it:*

  3. *changes the system dynamics*

- note that this is not without cost – in the example above, the voltage out of the velocity error amplifier becomes larger as $T_m^{'} \to 0$. Thus, improved response is at the cost of a larger error amplifier and increased power consumption.

14.2.3 Steady-state disturbance response

- consider now the steady state command response ($V_\Omega = 0$):

$$\left(\frac{\Omega_m}{T_l}\right)_{SS} = B' = \frac{B}{\left(K_v A K_t + 1\right)} \tag{14.17}$$

- thus, $\left(\frac{\Omega_m}{T_l}\right)_{SS} \to 0$ as $K_v A K_t \to \infty$

- *another important effect of feedback is therefore that it:*

  4. *can reduce the sensitivity of the system to disturbances in load ie. increased 'disturbance rejection'*