Lecture 13: Bode plots for systems

- we have now established the basic building blocks for constructing a Bode plot to represent the frequency response of a system. Each of these elements can be sketched rapidly and accurately using high, low and intermediate frequency asymptotes, and a few key points which define the "departure" of the actual curve from the asymptotes.

- then, as shown in the previous lecture, the Bode plot for a complete system, whose transfer function is the *product* of a number of elementary factors, may be found simply by adding the contributions of each element.

- for example, consider a system with the following transfer function:

$$G(s) = \frac{100(s+2)}{s\left[s^2+4s+100\right]} \qquad (13.1)$$

- which, in 'Bode form':

$$G(s) = \frac{2(\frac{s}{2}+1)}{s\left[\left(\frac{s}{10}\right)^2 + 2(0.2)\left(\frac{s}{10}\right)+1\right]} \qquad (13.2)$$

- we can now draw the asymptotes, and the departure from these asymptotes, for each of the factors:

1. $2/s$ - pole at the origin
   Note that it is convenient to combine the Bode gain and free-s factors into one factor. The amplitude curve is a straight line of slope $-1\ dec/dec$. The amplitude of $2/j\omega$ is unity when $\omega = 2$ rad/s; hence the straight line passes through the point $\omega = 2, A(\omega) = 1$. The phase of $2/j\omega$ is $\varphi(\omega) = -90°$ for all frequencies.

2. $(s/2+1)$ - first order lead
   This factor has a low-frequency amplitude asymptote $A(\omega) = 1$ (i.e., $Lm = 0$), a corner frequency of $\omega = 2$ rad/s, and a high-frequency asymptote with a slope of $+1\ dec/dec$. The mid-frequency phase asymptote passes through the point $\omega = 2,\ \varphi(\omega) = +45°$.

3. $\left[(s/10)^2 + 2(0.2)(s/10)+1\right]^{-1}$ - 2nd order lag
   In writing the 2nd order factor in this way, we identify the corner frequency (the undamped natural frequency) as 10 rad/s and the damping ratio $\zeta = 0.2$. The low-frequency amplitude asymptote is $A(\omega) = 1$, while the high-frequency asymptote is a line of slope $-2\ dec/dec$, passing through $\omega = 10, A(\omega) = 1$. For a damping ratio as small as 0.2, the departure from the asymptote can be

sketched accurately enough by noting that the amplitude ratio at the corner frequency is $Q = 1/2\zeta = 2.5$. The low- and high-frequency phase asymptotes are $0°$ and $-180°$ respectively. The mid-frequency asymptote passes through $-90°$ at the undamped natural frequency $\omega_n = 10$ rad/s.

- we then simply add the separate curves to get the amplitude ratio and phase curves for the complete system: