Appendix A

Laplace Transform Tables

Table A.1 Laplace Transform Properties and Theorems

Properties or theorems	$f(t)$	$F(s)$
1 Definition of the Laplace transform	$f(t)$	$\int_0^{\infty} f(t)e^{-st}\,\mathrm{d}t$
2 Definition of the inverse Laplace transform	$\frac{1}{2\pi j}\int_{c-j\omega}^{c+j\infty} F(s)\mathrm{e}^{st}\,\mathrm{d}s$	$F(s)$
3 Linearity	$c_1 f_1(t) + c_2 f_2(t)$	$c_1 F_1(s) + c_2 F_2(s)$
4 First derivative	$\frac{\mathrm{d}f(t)}{\mathrm{d}t}$	$sF(s) - f(0)$
5 Second derivative	$\frac{\mathrm{d}^2 f(t)}{\mathrm{d}t^2}$	$s^2 F(s) - sf(0) - f^{(1)}(0)$
6 nth derivative	$\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$
7 Integral	$\int_0^t f(t)\,\mathrm{d}t$	$\frac{F(s)}{s}$
8 Integral	$\int_{-\infty}^t f(t)\,\mathrm{d}t$	$\frac{F(s)}{s} + \frac{f^{(-1)}(0)}{s}$
9 Double integral	$\int_{-\infty}^t \int_{-\infty}^t f(t)(\mathrm{d}t)^2$	$\frac{F(s)}{s^2} + \frac{f^{(-1)}(0)}{s^2} + \frac{f^{(-2)}(0)}{s}$
10 nth time integral	$\underbrace{\int_{\infty}^t \cdots \int_{\infty}^t}_{n\ \text{times}} f(t)(\mathrm{d}t)^n$	$\frac{F(s)}{s^n} + \frac{f^{(-1)}(0)}{s^n} + \frac{f^{(-2)}(0)}{s^{n-1}} + \cdots + \frac{f^{(-n)}(0)}{s}$
11 Time scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$

(*continued*)

Table A.1 (*continued*)

Properties or theorems	$f(t)$	$F(s)$
12 Shift in the frequency domain	$e^{-at}f(t)$	$F(s+a)$
13 Shift in the time domain	$f(t-a)u(t-a)$	$e^{-at}F(s)$
14 Multiplication of a function by t	$tf(t)$	$-\frac{d}{ds}F(s)$
15 Division of a function by t	$\frac{f(t)}{t}$	$\int_s^\infty F(a)\,da$
16 Multiplication of a function by t^n	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$
17 Division of a function by t^n	$\frac{f(t)}{t^n}$	$\underbrace{\int_s^t \cdots \int_s^t}_{n\,\text{times}} F(s)(ds)^n$
18 Convolution	$\int_0^t h(t-\tau)u(\tau)d\tau$	$H(s)U(s)$
19 The initial value theorem	$\lim_{t\to 0} f(t)$	$\lim_{t\to\infty} sF(s)$
20 The final value theorem	$\lim_{t\to\infty} f(t)$	$\lim_{t\to 0} sF(s)$

Remark A.1.1 In the properties 8, 9, and 10, the constant $f^{(-k)}(0)$ is defined as follows:

$$f^{(-1)}(0) = \int_{-\infty}^0 f(t)\,dt,\ f^{(-2)}(0) = \int_{-\infty}^0 \int_{-\infty}^0 f(t)dt^2,\ \text{etc.}$$