Table A.2 Laplace Transform Pairs

SN	$F(s) = L[f(t)]$	$f(t) = L^{-1}[F(s)]$	Remarks
1	1	$\delta(t)$	
2	s	$\delta^{(1)}(t)$	
3	s^n	$\delta^{(n)}(t)$	n is a positive integer
4	$\dfrac{1}{s}$	$u(t)$	
5	$\dfrac{1}{s^2}$	t	
6	$\dfrac{1}{s^n}$	$\dfrac{t^{n-1}}{(n-1)!}$	n is a positive integer
7	$\dfrac{1}{s^{1/2}}$	$\dfrac{1}{(\pi t)^{1/2}}$	
8	$\dfrac{1}{s^{n+1/2}}$	$\dfrac{2^n t^{n-1/2}}{1 \cdot 3 \cdot 5 \cdots (2n-1) \cdot \pi^{1/2}}$	n is a positive integer
9	$\dfrac{1}{s+a}$	e^{-at}	
10	$\dfrac{1}{(s+a)^2}$	te^{-at}	
11	$\dfrac{1}{(s+a)^n}$	$\dfrac{t^{n-1}e^{-at}}{(n-1)!}$	n is a positive integer
12	$\dfrac{1}{s^2+a^2}$	$\dfrac{1}{a}\sin at$	
13	$\dfrac{1}{(s^2+a^2)^2}$	$\dfrac{1}{2a^3}(\sin at - at\cos at)$	
14	$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\sinh at$	
15	$\dfrac{1}{(s+a)(s+b)}$	$\dfrac{e^{-at}-e^{-bt}}{b-a}$	
16	$\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{-(c-b)e^{-at}-(a-c)e^{-bt}-(b-a)e^{-ct}}{(b-a)(c-b)(a-c)}$	
17	$\dfrac{(s+a)}{(s+b)(s+c)}$	$\dfrac{(a-b)e^{-bt}-(a-c)e^{-ct}}{(c-b)}$	
18	$\dfrac{s}{s^2+a^2}$	$\cos at$	

(*continued*)

Table A.2   (*continued*)

SN	$F(s) = L[f(t)]$	$f(t) = L^{-1}[F(s)]$	Remarks
19	$\dfrac{s}{s^2 - a^2}$	$\cosh at$	
20	$\dfrac{s + a}{(s + a)^2 + b^2}$	$e^{-at} \cos bt$	
21	$\dfrac{b}{(s + a)^2 + b^2}$	$e^{-at} \sin bt$	
22	$\dfrac{1}{s^2(s + a)}$	$\dfrac{1}{a^2}(e^{-at} + at - 1)$	
23	$\dfrac{1}{(s + a)^2(s + b)}$	$\dfrac{1}{(b - a)^2}\left[[(b - a)t - 1]e^{-at} + e^{-bt}\right]$	
24	$\dfrac{1}{s(s^2 + a^2)}$	$\dfrac{1}{a^2}(1 - \cos at)$	
25	$\dfrac{s}{(s + a)(s + b)}$	$\dfrac{1}{b - a}(be^{-bt} - ae^{-at})$	
26	$\dfrac{1}{s(s + a)^2}$	$\dfrac{1}{a^2}(1 - (at + 1)e^{-at})$	
27	$\dfrac{1}{s(s + a)(s + b)}$	$\dfrac{be^{-at} - ae^{-bt}}{ab(b - a)} + \dfrac{1}{ab}$	
28	$\dfrac{1}{s^2(s^2 + a^2)}$	$\dfrac{1}{a^3}(at - \sin at)$	
29	$\dfrac{1}{s^4 - a^4}$	$\dfrac{1}{2a^3}(\sinh at - \sin at)$	
30	$\dfrac{s + b}{s^2 + a^2}$	$\dfrac{\sqrt{a^2 + b^2}}{a} \sin(at + \theta)$	$\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$
31	$\dfrac{s}{(s^2 + a^2)^2}$	$\dfrac{t}{2a} \sin at$	
32	$\dfrac{s^2}{(s^2 + a^2)^2}$	$\dfrac{1}{2a}(at \cos at + \sin at)$	
33	$\dfrac{s^n}{(s^2 + a^2)^{n+1}}$	$\dfrac{t^n \sin at}{n!2^n a}$	
34	$\dfrac{s + b}{s(s + a)^2}$	$\dfrac{b}{a^2} + \left[\dfrac{a - b}{a}t - \dfrac{b}{a^2}e^{-at}\right]$	
35	$\dfrac{s + c}{s(s + a)(s + b)}$	$\dfrac{c - a}{a(a - b)}e^{-at} + \dfrac{c - b}{b(b - a)}e^{-bt} + \dfrac{c}{ab}$	

Table A.2 (*continued*)

SN	$F(s) = L[f(t)]$	$f(t) = L^{-1}[F(s)]$	Remarks
36	$\dfrac{s^2 - b^2}{(s^2 + b^2)^2}$	$t \cos bt$	
37	$\dfrac{s}{(s^2 + a^2)(s^2 + b^2)}$	$\dfrac{\cos at - \cos bt}{b^2 - a^2}$	$a \neq b$
38	$\dfrac{s}{(s+a)(s+b)(s+c)}$	$-\dfrac{ae^{-at}}{(b-a)(c-a)} - \dfrac{be^{-bt}}{(a-b)(c-b)}$ $-\dfrac{ce^{-ct}}{(a-c)(b-c)}$	
39	$\dfrac{s^2}{(s+a)(s+b)(s+c)}$	$\dfrac{a^2 e^{-at}}{(b-a)(c-a)} + \dfrac{b^2 e^{-bt}}{(a-b)(c-b)}$ $+\dfrac{c^2 e^{-ct}}{(a-c)(b-c)}$	
40	$\dfrac{s}{(s+a)(s+b)^2}$	$-\dfrac{ae^{-at} + (b(a-b)t - a)e^{-bt}}{(a-b)^2}$	
41	$\dfrac{s^2}{(s+a)(s+b)^2}$	$\dfrac{a^2 e^{-at} + (b^2(a-b)t + b^2 - 2ab)e^{-bt}}{(a-b)^2}$	
42	$\dfrac{1}{(s+a)(s^2 + b^2)^2}$	$\dfrac{1}{a^2 + b^2}\left[e^{-at} - \dfrac{1}{b}\sqrt{a^2 + b^2}\,\sin(bt + \theta)\right]$	$\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$
43	$\dfrac{s}{(s+a)(s^2 + b^2)}$	$\dfrac{-1}{a^2 + b^2}\left[e^{-at} - \dfrac{1}{a}\sqrt{a^2 + b^2}\,\sin(bt + \theta)\right]$	$\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$
44	$\dfrac{s^2}{(s+a)(s^2 + b^2)}$	$\dfrac{a^2}{a^2 + b^2}\left[e^{-at} - \dfrac{b}{a^2}\sqrt{a^2 + b^2}\,\sin(bt - \theta)\right]$	$\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$
45	$\dfrac{1}{s[(s+a)^2 + b^2]}$	$\dfrac{1}{a^2 + b^2}\left[1 - \dfrac{b}{a}\sqrt{a^2 + b^2}\,e^{-at}\sin(bt + \theta)\right]$	$\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$
46	$\dfrac{1}{s^2[(s+a)^2 + b^2]}$	$\dfrac{1}{a^2 + b^2}\left[t - \dfrac{2a}{a^2 + b^2} + \dfrac{1}{b}e^{-at}\sin(bt + \theta)\right]$	$\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$
47	$\dfrac{1}{s(s^2 + a^2)^2}$	$\dfrac{1}{a^4}(1 - \cos at) - \dfrac{1}{2a^3}t\sin at$	
48	$\dfrac{1}{s^4 - a^4}$	$\dfrac{1}{2a^2}(\cosh at - \cos at)$	
49	$\dfrac{s^2}{s^4 - a^4}$	$\dfrac{1}{2a}(\sinh at + \sin at)$	

(*continued*)

Table A.2   (*continued*)

SN	$F(s) = L[f(t)]$	$f(t) = L^{-1}[F(s)]$	Remarks
50	$\dfrac{1}{\sqrt{s^2 + a^2}}$	$J_0(at)$	J_0: Bessel function
51	$\dfrac{1}{s - \ln b}$	b^t	
52	$\ln \dfrac{s + a}{s + b}$	$\dfrac{1}{t}(e^{-bt} - e^{-at})$	