

for stability. You will also be able to find percent overshoot, settling time, peak time, and rise time, given K .

10.1 Introduction

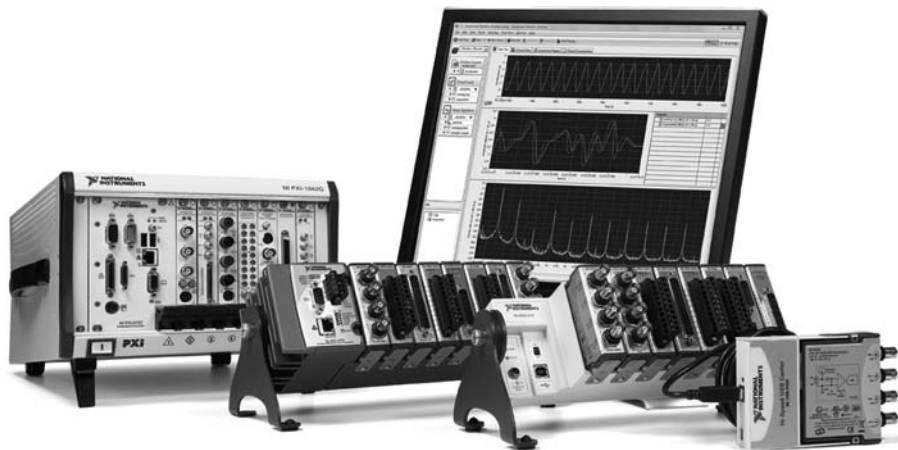
The root locus method for transient design, steady-state design, and stability was covered in Chapters 8 and 9. In Chapter 8, we covered the simple case of design through gain adjustment, where a trade-off was made between a desired transient response and a desired steady-state error. In Chapter 9, the need for this trade-off was eliminated by using compensation networks so that transient and steady-state errors could be separately specified and designed. Further, a desired transient response no longer had to be on the original system's root locus.

This chapter and Chapter 11 present the design of feedback control systems through gain adjustment and compensation networks from another perspective—that of frequency response. The results of frequency response compensation techniques are not new or different from the results of root locus techniques.

Frequency response methods, developed by Nyquist and Bode in the 1930s, are older than the root locus method, which was discovered by Evans in 1948 (*Nyquist, 1932; Bode, 1945*). The older method, which is covered in this chapter, is not as intuitive as the root locus. However, frequency response yields a new vantage point from which to view feedback control systems. This technique has distinct advantages in the following situations:

1. When modeling transfer functions from physical data, as shown in Figure 10.1
2. When designing lead compensators to meet a steady-state error requirement and a transient response requirement
3. When finding the stability of nonlinear systems
4. In settling ambiguities when sketching a root locus

FIGURE 10.1 National Instruments PXI, Compact RIO, Compact DAQ, and USB hardware platforms (shown from left to right) couple with NI LabVIEW software to provide stimulus and acquire signals from physical systems. NI LabVIEW can then be used to analyze data, determine the mathematical model, and prototype and deploy a controller for the physical system (Courtesy National Instruments © 2010).



We first discuss the concept of frequency response, define frequency response, derive analytical expressions for the frequency response, plot the frequency response, develop ways of sketching the frequency response, and then apply the concept to control system analysis and design.

The Concept of Frequency Response

In the steady state, sinusoidal inputs to a linear system generate sinusoidal responses of the same frequency. Even though these responses are of the same frequency as the input, they differ in amplitude and phase angle from the input. These differences are functions of frequency.

Before defining frequency response, let us look at a convenient representation of sinusoids. Sinusoids can be represented as complex numbers called *phasors*. The magnitude of the complex number is the amplitude of the sinusoid, and the angle of the complex number is the phase angle of the sinusoid. Thus, $M_1 \cos(\omega t + \phi_1)$ can be represented as $M_1 \angle \phi_1$ where the frequency, ω , is implicit.

Since a system causes both the amplitude and phase angle of the input to be changed, we can think of the system itself as represented by a complex number, defined so that the product of the input phasor and the system function yields the phasor representation of the output.

Consider the mechanical system of Figure 10.2(a). If the input force, $f(t)$, is sinusoidal, the steady-state output response, $x(t)$, of the system is also sinusoidal and at the same frequency as the input. In Figure 10.2(b) the input and output sinusoids are represented by complex numbers, or phasors, $M_i(\omega) \angle \phi_i(\omega)$ and $M_o(\omega) \angle \phi_o(\omega)$, respectively. Here the M 's are the amplitudes of the sinusoids, and the ϕ 's are the phase angles

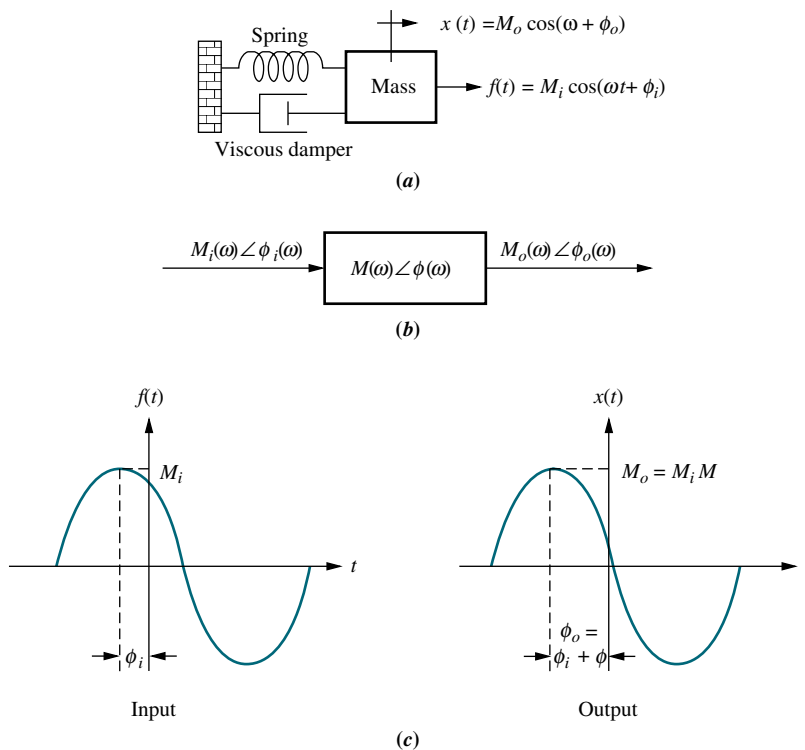


FIGURE 10.2 Sinusoidal frequency response: **a.** system; **b.** transfer function; **c.** input and output waveforms

of the sinusoids as shown in Figure 10.2(c). Assume that the system is represented by the complex number, $M(\omega)\angle\phi(\omega)$. The output steady-state sinusoid is found by multiplying the complex number representation of the input by the complex number representation of the system. Thus, the steady-state output sinusoid is

$$M_o(\omega)\angle\phi_o(\omega) = M_i(\omega)M(\omega)\angle[\phi_i(\omega) + \phi(\omega)] \quad (10.1)$$

From Eq. (10.1) we see that the system function is given by

$$M(\omega) = \frac{M_o(\omega)}{M_i(\omega)} \quad (10.2)$$

and

$$\phi(\omega) = \phi_o(\omega) - \phi_i(\omega) \quad (10.3)$$

Equations (10.2) and (10.3) form our definition of frequency response. We call $M(\omega)$ the *magnitude frequency response* and $\phi(\omega)$ the *phase frequency response*. The combination of the magnitude and phase frequency responses is called the *frequency response* and is $M(\omega)\angle\phi(\omega)$.

In other words, we define the magnitude frequency response to be the ratio of the output sinusoid's magnitude to the input sinusoid's magnitude. We define the phase response to be the difference in phase angle between the output and the input sinusoids. Both responses are a function of frequency and apply only to the steady-state sinusoidal response of the system.

Analytical Expressions for Frequency Response

Now that we have defined frequency response, let us obtain the analytical expression for it (Nilsson, 1990). Later in the chapter, we will use this analytical expression to determine stability, transient response, and steady-state error. Figure 10.3 shows a system, $G(s)$, with the Laplace transform of a general sinusoid, $r(t) = A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \cos[\omega t - \tan^{-1}(B/A)]$ as the input. We can represent the input as a phasor in three ways: (1) in polar form, $M_i\angle\phi_i$, where $M_i = \sqrt{A^2 + B^2}$ and $\phi_i = -\tan^{-1}(B/A)$; (2) in rectangular form, $A - jB$; and (3) using Euler's formula, $M_i e^{j\phi_i}$.

We now solve for the forced response portion of $C(s)$, from which we evaluate the frequency response. From Figure 10.3,

$$C(s) = \frac{As + B\omega}{(s^2 + \omega^2)} G(s) \quad (10.4)$$

We separate the forced solution from the transient solution by performing a partial-fraction expansion on Eq. (10.4). Thus,

$$\begin{aligned} C(s) &= \frac{As + B\omega}{(s + j\omega)(s - j\omega)} G(s) \\ &= \frac{K_1}{s + j\omega} + \frac{K_2}{s - j\omega} + \text{Partial fraction terms from } G(s) \end{aligned} \quad (10.5)$$

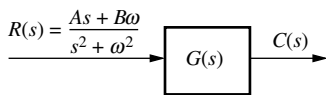


FIGURE 10.3 System with sinusoidal input

where

$$K_1 = \frac{As + B\omega}{s - j\omega} G(s) \Big|_{s \rightarrow -j\omega} = \frac{1}{2}(A + jB)G(-j\omega) = \frac{1}{2}M_i e^{-j\phi_i} M_G e^{-j\phi_G} \\ = \frac{M_i M_G}{2} e^{-j(\phi_i + \phi_G)} \quad (10.6a)$$

$$K_2 = \frac{As + B\omega}{s + j\omega} G(s) \Big|_{s \rightarrow +j\omega} = \frac{1}{2}(A - jB)G(j\omega) = \frac{1}{2}M_i e^{j\phi_i} M_G e^{j\phi_G} \\ = \frac{M_i M_G}{2} e^{j(\phi_i + \phi_G)} = K_1^* \quad (10.6b)$$

For Eqs. (10.6), K_1^* is the complex conjugate of K_1 ,

$$M_G = |G(j\omega)| \quad (10.7)$$

and

$$\phi_G = \text{angle of } G(j\omega) \quad (10.8)$$

The steady-state response is that portion of the partial-fraction expansion that comes from the input waveform's poles, or just the first two terms of Eq. (10.5). Hence, the sinusoidal steady-state output, $C_{ss}(s)$, is

$$C_{ss}(s) = \frac{K_1}{s + j\omega} + \frac{K_2}{s - j\omega} \quad (10.9)$$

Substituting Eqs. (10.6) into Eq. (10.9), we obtain

$$C_{ss}(s) = \frac{\frac{M_i M_G}{2} e^{-j(\phi_i + \phi_G)}}{s + j\omega} + \frac{\frac{M_i M_G}{2} e^{j(\phi_i + \phi_G)}}{s - j\omega} \quad (10.10)$$

Taking the inverse Laplace transformation, we obtain

$$c(t) = M_i M_G \left(\frac{e^{-j(\omega t + \phi_i + \phi_G)} + e^{j(\omega t + \phi_i + \phi_G)}}{2} \right) \\ = M_i M_G \cos(\omega t + \phi_i + \phi_G) \quad (10.11)$$

which can be represented in phasor form as $M_o \angle \phi_o = (M_1 \angle \phi_1)(M_G \angle \phi_G)$, where $M_G \angle \phi_G$ is the frequency response function. But from Eqs. (10.7) and (10.8), $M_G \angle \phi_G = G(j\omega)$. In other words, the frequency response of a system whose transfer function is $G(s)$ is

$$G(j\omega) = G(s) \Big|_{s \rightarrow j\omega} \quad (10.12)$$

Plotting Frequency Response

$G(j\omega) = M_G(\omega) \angle \phi_G(\omega)$ can be plotted in several ways; two of them are (1) as a function of frequency, with separate magnitude and phase plots; and (2) as a polar plot, where the phasor length is the magnitude and the phasor angle is the phase. When plotting separate magnitude and phase plots, the magnitude curve can be plotted in

decibels (dB) vs. $\log \omega$, where $\text{dB} = 20 \log M$.¹ The phase curve is plotted as phase angle vs. $\log \omega$. The motivation for these plots is shown in Section 10.2.

Using the concepts covered in Section 8.1, data for the plots also can be obtained using vectors on the s -plane drawn from the poles and zeros of $G(s)$ to the imaginary axis. Here the magnitude response at a particular frequency is the product of the vector lengths from the zeros of $G(s)$ divided by the product of the vector lengths from the poles of $G(s)$ drawn to points on the imaginary axis. The phase response is the sum of the angles from the zeros of $G(s)$ minus the sum of the angles from the poles of $G(s)$ drawn to points on the imaginary axis. Performing this operation for successive points along the imaginary axis yields the data for the frequency response. Remember, each point is equivalent to substituting that point, $s = j\omega_1$, into $G(s)$ and evaluating its value.

The plots also can be made from a computer program that calculates the frequency response. For example, the root locus program discussed in Appendix H at www.wiley.com/college/nise can be used with test points that are on the imaginary axis. The calculated K value at each frequency is the reciprocal of the scaled magnitude response, and the calculated angle is, directly, the phase angle response at that frequency.

The following example demonstrates how to obtain an analytical expression for frequency response and make a plot of the result.

Example 10.1

Frequency Response from The Transfer Function

PROBLEM: Find the analytical expression for the magnitude frequency response and the phase frequency response for a system $G(s) = 1/(s + 2)$. Also, plot both the separate magnitude and phase diagrams and the polar plot.

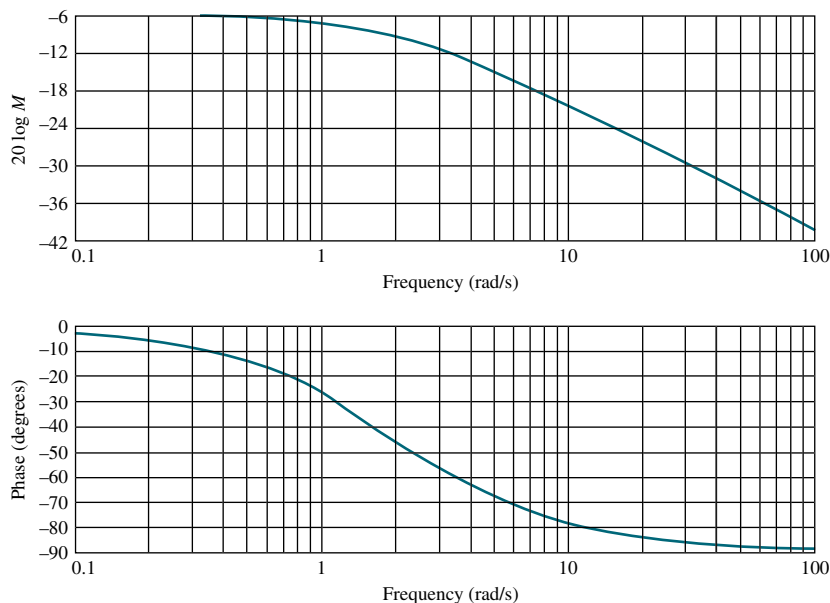


FIGURE 10.4 Frequency response plots for $G(s) = 1/(s + 2)$: separate magnitude and phase diagrams.

¹Throughout this book, “log” is used to mean \log_{10} , or logarithm to the base 10.

SOLUTION: First substitute $s = j\omega$ in the system function and obtain $G(j\omega) = 1/(j\omega + 2) = (2 - j\omega)/(\omega^2 + 4)$. The magnitude of this complex number, $|G(j\omega)| = M(\omega) = 1/\sqrt{\omega^2 + 4}$, is the magnitude frequency response. The phase angle of $G(j\omega)$, $\phi(\omega) = -\tan^{-1}(\omega/2)$, is the phase frequency response.

$G(j\omega)$ can be plotted in two ways: (1) in separate magnitude and phase plots and (2) in a polar plot. Figure 10.4 shows separate magnitude and phase diagrams, where the magnitude diagram is $20 \log M(\omega) = 20 \log (1/\sqrt{\omega^2 + 4})$ vs. $\log \omega$, and the phase diagram is $\phi(\omega) = -\tan^{-1}(\omega/2)$ vs. $\log \omega$. The polar plot, shown in Figure 10.5, is a plot of $M(\omega) \angle \phi(\omega) = 1/\sqrt{\omega^2 + 4} \angle -\tan^{-1}(\omega/2)$ for different ω .

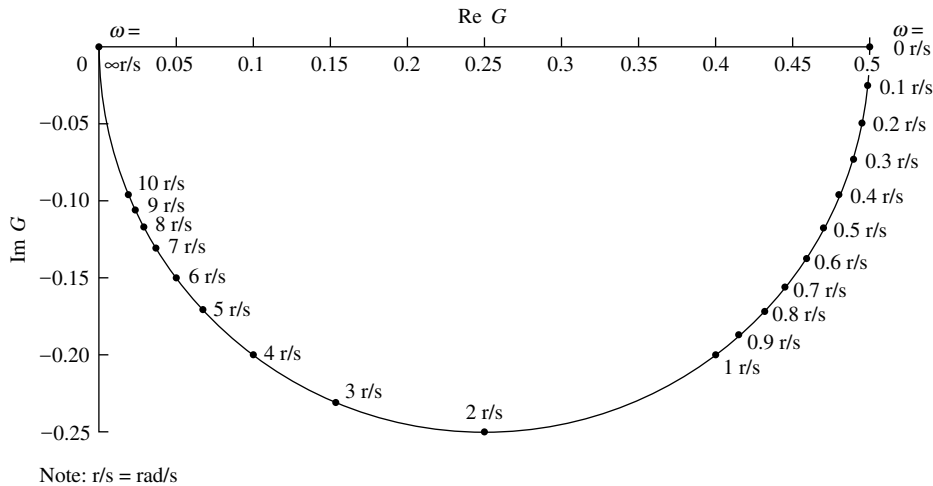


FIGURE 10.5 Frequency response plot for $G(s) = 1/(s + 2)$: polar plot

In the previous example, we plotted the separate magnitude and phase responses, as well as the polar plot, using the mathematical expression for the frequency response. Either of these frequency response presentations can also be obtained from the other. You should practice this conversion by looking at Figure 10.4 and obtaining Figure 10.5 using successive points. For example, at a frequency of 1 rad/s in Figure 10.4, the magnitude is approximately -7 dB, or $10^{-7/20} = 0.447$. The phase plot at 1 rad/s tells us that the phase is about -26° . Thus, on the polar plot a point of radius 0.447 at an angle of -26° is plotted and identified as 1 rad/s. Continuing in like manner for other frequencies in Figure 10.4, you can obtain Figure 10.5.

Similarly, Figure 10.4 can be obtained from Figure 10.5 by selecting a sequence of points in Figure 10.5 and translating them to separate magnitude and phase values. For example, drawing a vector from the origin to the point at 2 rad/s in Figure 10.5, we see that the magnitude is $20 \log 0.35 = -9.12$ dB and the phase angle is about -45° . The magnitude and phase angle are then plotted at 2 rad/s in Figure 10.4 on the separate magnitude and phase curves.

Skill-Assessment Exercise 10.1

PROBLEM:

- a. Find analytical expressions for the magnitude and phase responses of

$$G(s) = \frac{1}{(s + 2)(s + 4)}$$

- b. Make plots of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate.
- c. Make a polar plot of the frequency response.

ANSWERS:

$$\text{a. } M(\omega) = \frac{1}{\sqrt{(8 - \omega^2)^2 + (6\omega)^2}}; \text{ for } \omega \leq \sqrt{8}: \phi(\omega) = -\arctan\left(\frac{6\omega}{8 - \omega^2}\right), \text{ for}$$

$$\omega > \sqrt{8}: \phi(\omega) = -\left[\pi + \arctan\left(\frac{6\omega}{8 - \omega^2}\right)\right]$$

b. See the answer at www.wiley.com/college/nise.

c. See the answer at www.wiley.com/college/nise.

The complete solution is at www.wiley.com/college/nise.

In this section, we defined frequency response and saw how to obtain an analytical expression for the frequency response of a system simply by substituting $s = j\omega$ into $G(s)$. We also saw how to make a plot of $G(j\omega)$. The next section shows how to approximate the magnitude and phase plots in order to sketch them rapidly.

10.2 Asymptotic Approximations: Bode Plots

The log-magnitude and phase frequency response curves as functions of $\log \omega$ are called Bode plots or Bode diagrams. Sketching Bode plots can be simplified because they can be approximated as a sequence of straight lines. Straight-line approximations simplify the evaluation of the magnitude and phase frequency response.

Consider the following transfer function:

$$G(s) = \frac{K(s + z_1)(s + z_2) \cdots (s + z_k)}{s^m(s + p_1)(s + p_2) \cdots (s + p_n)} \quad (10.13)$$

The magnitude frequency response is the product of the magnitude frequency responses of each term, or

$$|G(j\omega)| = \frac{K|(s + z_1)||s + z_2| \cdots |s + z_k|}{|s^m|(s + p_1)|(s + p_2)| \cdots |(s + p_n)|} \Big|_{s \rightarrow j\omega} \quad (10.14)$$

Thus, if we know the magnitude response of each pole and zero term, we can find the total magnitude response. The process can be simplified by working with the logarithm of the magnitude since the zero terms' magnitude responses would be added and the pole terms' magnitude responses subtracted, rather than, respectively, multiplied or divided, to yield the logarithm of the total magnitude response. Converting the magnitude response into dB, we obtain

$$20 \log |G(j\omega)| = 20 \log K + 20 \log |(s + z_1)| + 20 \log |(s + z_2)|$$

$$+ \cdots - 20 \log |s^m| - 20 \log |(s + p_1)| - \cdots \Big|_{s \rightarrow j\omega} \quad (10.15)$$

Thus, if we knew the response of each term, the algebraic sum would yield the total response in dB. Further, if we could make an approximation of each term that would consist only of straight lines, graphical addition of terms would be greatly simplified.

Before proceeding, let us look at the phase response. From Eq. (10.13), the phase frequency response is the *sum* of the phase frequency response curves of the zero terms minus the *sum* of the phase frequency response curves of the pole terms. Again, since the phase response is the sum of individual terms, straight-line approximations to these individual responses simplify graphical addition.

Let us now show how to approximate the frequency response of simple pole and zero terms by straight-line approximations. Later we show how to combine these responses to sketch the frequency response of more complicated functions. In subsequent sections, after a discussion of the Nyquist stability criterion, we learn how to use the Bode plots for the analysis and design of stability and transient response.

Bode Plots for $G(s) = (s + a)$

Consider a function, $G(s) = (s + a)$, for which we want to sketch separate logarithmic magnitude and phase response plots. Letting $s = j\omega$, we have

$$G(j\omega) = (j\omega + a) = a\left(j\frac{\omega}{a} + 1\right) \quad (10.16)$$

At low frequencies when ω approaches zero,

$$G(j\omega) \approx a \quad (10.17)$$

The magnitude response in dB is

$$20 \log M = 20 \log a \quad (10.18)$$

where $M = |G(j\omega)|$ and is a constant. Eq. (10.18) is shown plotted in Figure 10.6(a) from $\omega = 0.01a$ to a .

At high frequencies where $\omega \gg a$, Eq. (10.16) becomes

$$G(j\omega) \approx a\left(\frac{j\omega}{a}\right) = a\left(\frac{\omega}{a}\right) \angle 90^\circ = \omega \angle 90^\circ \quad (10.19)$$

The magnitude response in dB is

$$20 \log M = 20 \log a + 20 \log \frac{\omega}{a} = 20 \log \omega \quad (10.20)$$

where $a < \omega < \infty$. Notice from the middle term that the high-frequency approximation is equal to the low-frequency approximation when $\omega = a$, and increases for $\omega > a$.

If we plot dB, $20 \log M$, against $\log \omega$, Eq. (10.20) becomes a straight line:

$$y = 20x \quad (10.21)$$

where $y = 20 \log M$, and $x = \log \omega$. The line has a slope of 20 when plotted as dB vs. $\log \omega$.

Since each doubling of frequency causes $20 \log \omega$ to increase by 6 dB, the line rises at an equivalent slope of 6 dB/octave, where an *octave* is a doubling of frequency. This rise begins at $\omega = a$, where the low-frequency approximation equals the high-frequency approximation.

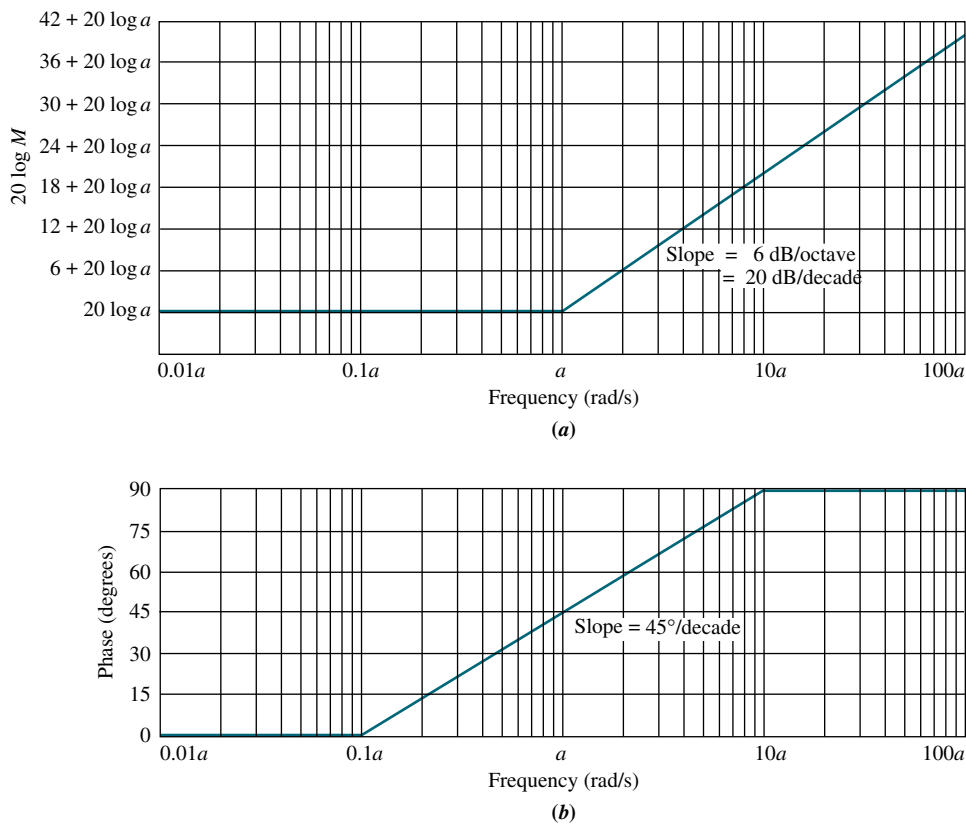


FIGURE 10.6 Bode plots of $(s + a)$: **a.** magnitude plot; **b.** phase plot

We call the straight-line approximations *asymptotes*. The low-frequency approximation is called the *low-frequency asymptote*, and the high-frequency approximation is called the *high-frequency asymptote*. The frequency, a , is called the *break frequency* because it is the break between the low- and the high-frequency asymptotes.

Many times it is convenient to draw the line over a decade rather than an octave, where a *decade* is 10 times the initial frequency. Over one decade, $20 \log \omega$ increases by 20 dB. Thus, a slope of 6 dB/octave is equivalent to a slope of 20 dB/decade. The plot is shown in Figure 10.6(a) from $\omega = 0.01a$ to $100a$.

Let us now turn to the phase response, which can be drawn as follows. At the break frequency, a , Eq. (10.16) shows the phase to be 45° . At low frequencies, Eq. (10.17) shows that the phase is 0° . At high frequencies, Eq. (10.19) shows that the phase is 90° . To draw the curve, start one decade ($1/10$) below the break frequency, $0.1a$, with 0° phase, and draw a line of slope $+45^\circ/\text{decade}$ passing through 45° at the break frequency and continuing to 90° one decade above the break frequency, $10a$. The resulting phase diagram is shown in Figure 10.6(b).

It is often convenient to *normalize* the magnitude and *scale* the frequency so that the log-magnitude plot will be 0 dB at a break frequency of unity. Normalizing and scaling helps in the following applications:

1. When comparing different first- or second-order frequency response plots, each plot will have the same low-frequency asymptote after normalization and the same break frequency after scaling.

2. When sketching the frequency response of a function such as Eq. (10.13), each factor in the numerator and denominator will have the same low-frequency asymptote after normalization. This common low-frequency asymptote makes it easier to add components to obtain the Bode plot.

To normalize $(s + a)$, we factor out the quantity a and form $a[(s/a) + 1]$. The frequency is scaled by defining a new frequency variable, $s_1 = s/a$. Then the magnitude is divided by the quantity a to yield 0 dB at the break frequency. Hence, the normalized and scaled function is $(s_1 + 1)$. To obtain the original frequency response, the magnitude and frequency are multiplied by the quantity a .

We now use the concepts of normalization and scaling to compare the asymptotic approximation to the actual magnitude and phase plot for $(s + a)$. Table 10.1 shows the comparison for the normalized and scaled frequency response of $(s + a)$. Notice that the actual magnitude curve is never greater than 3.01 dB from the asymptotes. This maximum difference occurs at the break frequency. The maximum difference for the phase curve is 5.71° , which occurs at the decades above and below the break frequency. For convenience, the data in Table 10.1 is plotted in Figures 10.7 and 10.8.

We now find the Bode plots for other common transfer functions.

TABLE 10.1 Asymptotic and actual normalized and scaled frequency response data for $(s + a)$

Frequency a (rad/s)	$20 \log \frac{M}{a}$ (dB)		Phase (degrees)	
	Asymptotic	Actual	Asymptotic	Actual
0.01	0	0.00	0.00	0.57
0.02	0	0.00	0.00	1.15
0.04	0	0.01	0.00	2.29
0.06	0	0.02	0.00	3.43
0.08	0	0.03	0.00	4.57
0.1	0	0.04	0.00	5.71
0.2	0	0.17	13.55	11.31
0.4	0	0.64	27.09	21.80
0.6	0	1.34	35.02	30.96
0.8	0	2.15	40.64	38.66
1	0	3.01	45.00	45.00
2	6	6.99	58.55	63.43
4	12	12.30	72.09	75.96
6	15.56	15.68	80.02	80.54
8	18	18.13	85.64	82.87
10	20	20.04	90.00	84.29
20	26.02	26.03	90.00	87.14
40	32.04	32.04	90.00	88.57
60	35.56	35.56	90.00	89.05
80	38.06	38.06	90.00	89.28
100	40	40.00	90.00	89.43

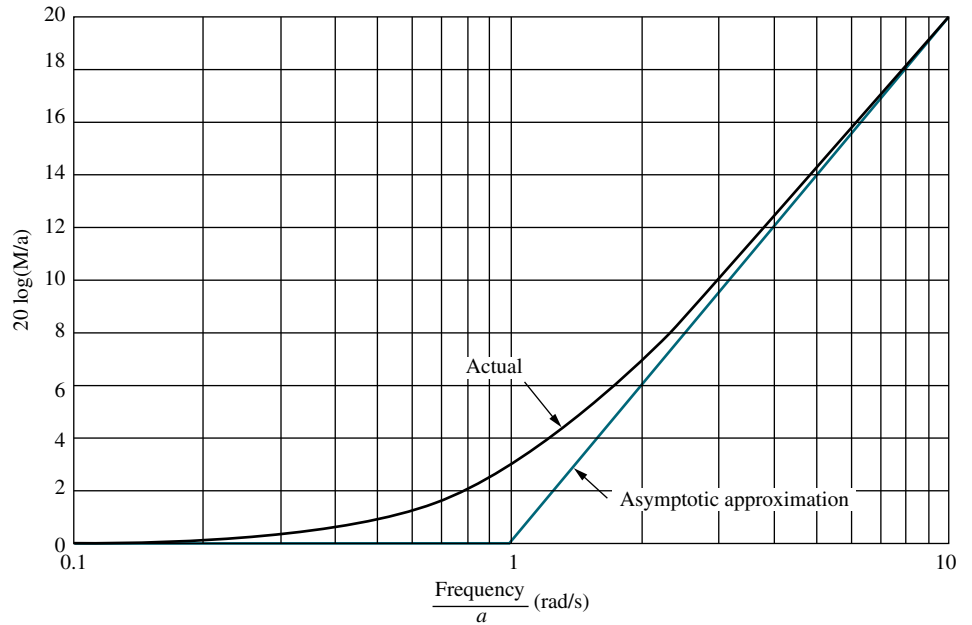


FIGURE 10.7 Asymptotic and actual normalized and scaled magnitude response of $(s + a)$

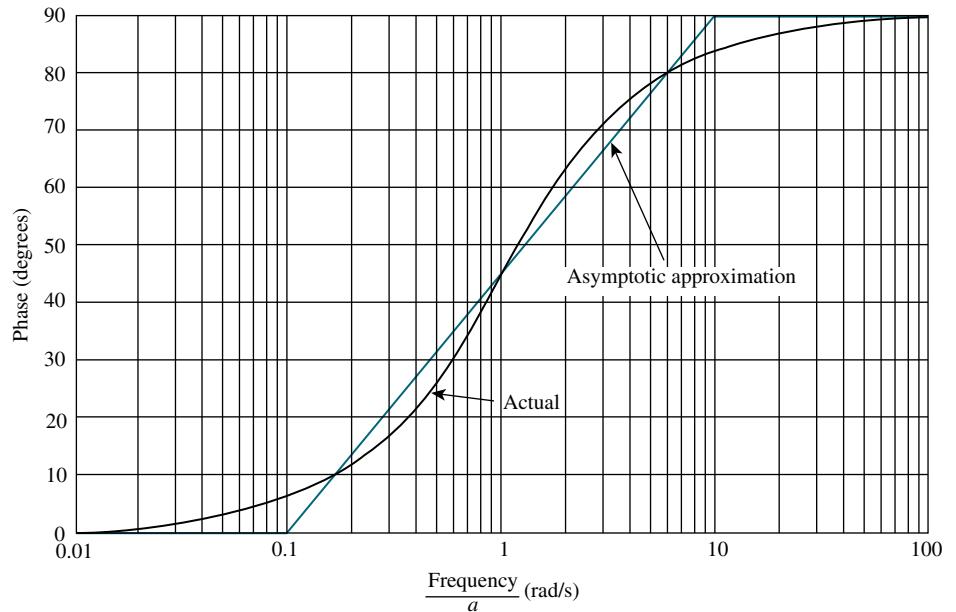


FIGURE 10.8 Asymptotic and actual normalized and scaled phase response of $(s + a)$

Bode Plots for $G(s) = 1/(s+a)$

Let us find the Bode plots for the transfer function

$$G(s) = \frac{1}{(s+a)} = \frac{1}{a\left(\frac{s}{a} + 1\right)} \quad (10.22)$$

This function has a low-frequency asymptote of $20 \log(1/a)$, which is found by letting the frequency, s , approach zero. The Bode plot is constant until the break frequency, a rad/s, is reached. The plot is then approximated by the high-frequency asymptote found by letting s approach ∞ . Thus, at high frequencies

$$G(j\omega) = \frac{1}{a\left(\frac{s}{a}\right)} \Big|_{s \rightarrow j\omega} = \frac{1}{a\left(\frac{j\omega}{a}\right)} = \frac{1}{\frac{a}{\omega} \angle -90^\circ} = \frac{1}{\omega} \angle -90^\circ \quad (10.23)$$

or, in dB,

$$20 \log M = 20 \log \frac{1}{a} - 20 \log \frac{\omega}{a} = -20 \log \omega \quad (10.24)$$

Notice from the middle term that the high-frequency approximation equals the low-frequency approximation when $\omega = a$, and decreases for $\omega > a$. This result is similar to Eq. (10.20), except the slope is negative rather than positive. The Bode log-magnitude diagram will decrease at a rate of 20 dB/decade rather than increase at a rate of 20 dB/decade after the break frequency.

The phase plot is the negative of the previous example since the function is the inverse. The phase begins at 0° and reaches -90° at high frequencies, going through -45° at the break frequency. Both the Bode normalized and scaled log-magnitude and phase plot are shown in Figure 10.9(d).

Bode Plots for $G(s) = s$

Our next function, $G(s) = s$, has only a high-frequency asymptote. Letting $s = j\omega$, the magnitude is $20 \log \omega$, which is the same as Eq. (10.20). Hence, the Bode magnitude plot is a straight line drawn with a +20 dB/decade slope passing through zero dB when $\omega = 1$. The phase plot, which is a constant $+90^\circ$, is shown with the magnitude plot in Figure 10.9(a).

Bode Plots for $G(s) = 1/s$

The frequency response of the inverse of the preceding function, $G(s) = 1/s$, is shown in Figure 10.9(b) and is a straight line with a -20 dB/decade slope passing through zero dB at $\omega = 1$. The Bode phase plot is equal to a constant -90° .

We have covered four functions that have first-order polynomials in s in the numerator or denominator. Before proceeding to second-order polynomials, let us

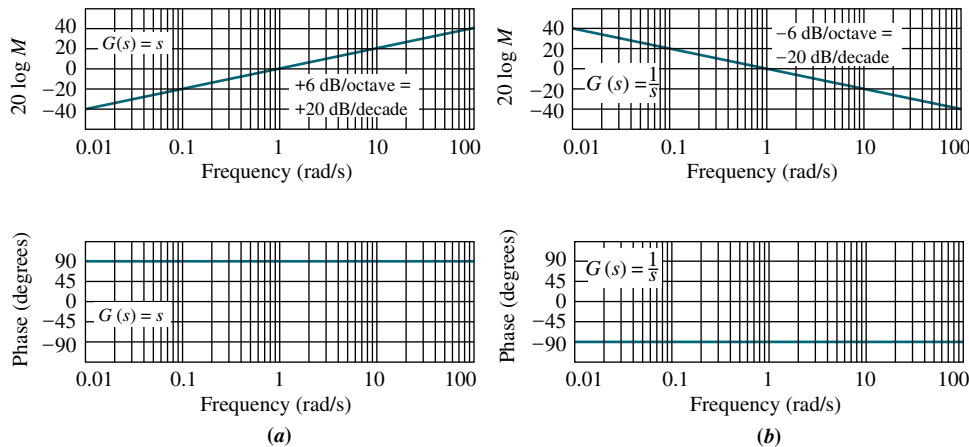


FIGURE 10.9 Normalized and scaled Bode plots for **a.** $G(s) = s$; **b.** $G(s) = 1/s$; (figure continues)

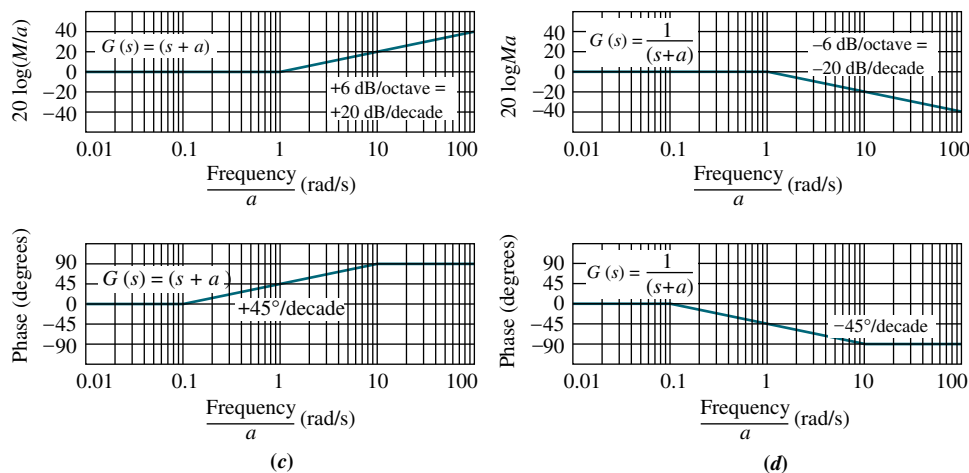


FIGURE 10.9 (Continued)
c. $G(s) = (s + a)$;
d. $G(s) = 1/(s + a)$

look at an example of drawing the Bode plots for a function that consists of the product of first-order polynomials in the numerator and denominator. The plots will be made by adding together the individual frequency response curves.

Example 10.2

Bode Plots for Ratio of First-Order Factors

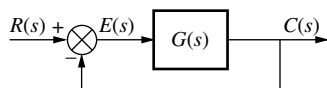


FIGURE 10.10 Closed-loop unity feedback system

PROBLEM: Draw the Bode plots for the system shown in Figure 10.10, where $G(s) = K(s + 3)/[s(s + 1)(s + 2)]$.

SOLUTION: We will make a Bode plot for the open-loop function $G(s) = K(s + 3)/[s(s + 1)(s + 2)]$. The Bode plot is the sum of the Bode plots for each first-order term. Thus, it is convenient to use the normalized plot for each of these terms so that the low-frequency asymptote of each term, except the pole at the origin, is at 0 dB, making it easier to add the components of the Bode plot. We rewrite $G(s)$ showing each term normalized to a low-frequency gain of unity. Hence,

$$G(s) = \frac{\frac{3}{2}K\left(\frac{s}{3} + 1\right)}{s(s + 1)\left(\frac{s}{2} + 1\right)} \tag{10.25}$$

Now determine that the break frequencies are at 1, 2, and 3. The magnitude plot should begin a decade below the lowest break frequency and extend a decade above the highest break frequency. Hence, we choose 0.1 radian to 100 radians, or three decades, as the extent of our plot.

At $\omega = 0.1$ the low-frequency value of the function is found from Eq. (10.25) using the low-frequency values for all of the $[(s/a) + 1]$ terms, (that is, $s = 0$) and the actual value for the s term in the denominator. Thus, $G(j0.1) \approx \frac{3}{2}K/0.1 = 15K$. The effect of K is to move the magnitude curve up (increasing K) or down (decreasing K) by the amount of $20 \log K$. K has no effect upon the phase curve. If we choose $K = 1$, the magnitude plot can be denormalized later for any value of K that is calculated or known.

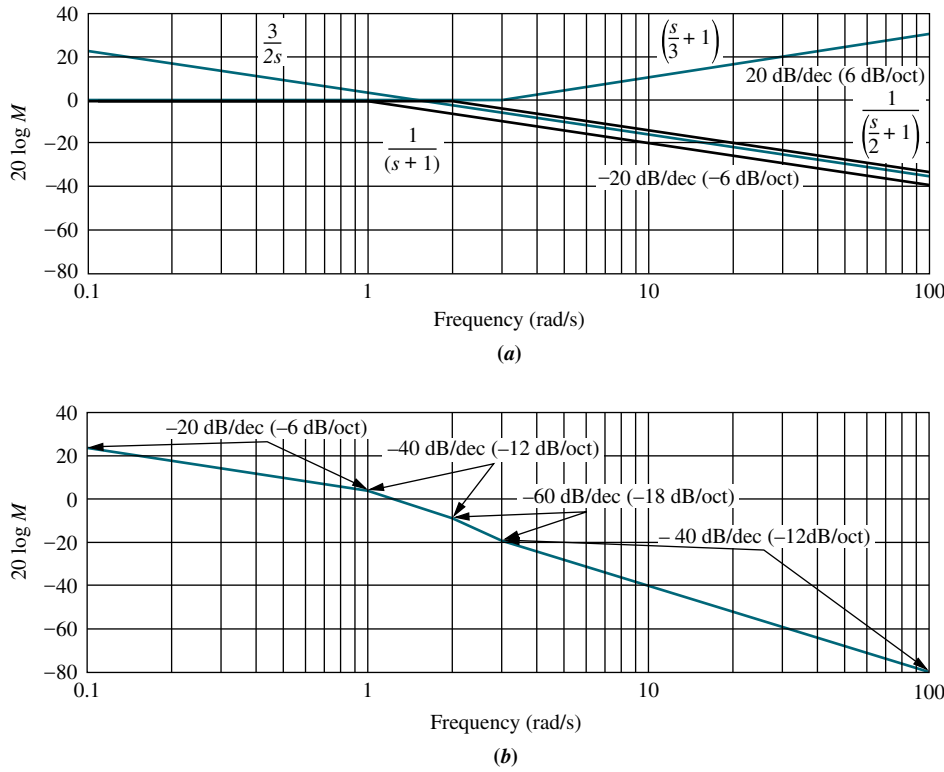


FIGURE 10.11 Bode log-magnitude plot for Example 10.2: **a.** components; **b.** composite

Figure 10.11(a) shows each component of the Bode log-magnitude frequency response. Summing the components yields the composite plot shown in Figure 10.11(b). The results are summarized in Table 10.2, which can be used to obtain the slopes. Each pole and zero is itemized in the first column. Reading across the table shows its contribution at each frequency. The last row is the sum of the slopes and correlates with Figure 10.11(b). The Bode magnitude plot for $K = 1$ starts at $\omega = 0.1$ with a value of $20 \log 15 = 23.52$ dB, and decreases immediately at a rate of -20 dB/decade, due to the s term in the denominator. At $\omega = 1$, the $(s + 1)$ term in the denominator begins its 20 dB/decade downward slope and causes an additional 20 dB/decade negative slope, or a total of -40 dB/decade. At $\omega = 2$, the term $[(s/2) + 1]$ begins its -20 dB/decade slope, adding yet another -20 dB/decade to the resultant plot, or a total of -60 dB/decade slope that continues until $\omega = 3$. At this frequency, the $[(s/3) + 1]$ term in the numerator begins its positive

TABLE 10.2 Bode magnitude plot: slope contribution from each pole and zero in Example 10.2

Description	Frequency (rad/s)			
	0.1 (Start: Pole at 0)	1 (Start: Pole at -1)	2 (Start: Pole at -2)	3 (Start: Zero at -3)
Pole at 0	-20	-20	-20	-20
Pole at -1	0	-20	-20	-20
Pole at -2	0	0	-20	-20
Zero at -3	0	0	0	20
Total slope (dB/dec)	-20	-40	-60	-40

20 dB/decade slope. The resultant magnitude plot, therefore, changes from a slope of -60 dB/decade to -40 dB/decade at $\omega = 3$, and continues at that slope since there are no other break frequencies.

The slopes are easily drawn by sketching straight-line segments decreasing by 20 dB over a decade. For example, the initial -20 dB/decade slope is drawn from 23.52 dB at $\omega = 0.1$, to 3.52 dB (a 20 dB decrease) at $\omega = 1$. The -40 dB/decade slope starting at $\omega = 1$ is drawn by sketching a line segment from 3.52 dB at $\omega = 1$, to -36.48 dB (a 40 dB decrease) at $\omega = 10$, and using only the portion from $\omega = 1$ to $\omega = 2$. The next slope of -60 dB/decade is drawn by first sketching a line segment from $\omega = 2$ to $\omega = 20$ (1 decade) that drops down by 60 dB, and using only that portion of the line from $\omega = 2$ to $\omega = 3$. The final slope is drawn by sketching a line segment from $\omega = 3$ to $\omega = 30$ (1 decade) that drops by 40 dB. This slope continues to the end of the plot.

Phase is handled similarly. However, the existence of breaks a decade below and a decade above the break frequency requires a little more bookkeeping. Table 10.3 shows the starting and stopping frequencies of the 45° /decade slope for

TABLE 10.3 Bode phase plot: slope contribution from each pole and zero in Example 10.2

Description	Frequency (rad/s)					
	0.1 (Start: Pole at -1)	0.2 (Start: Pole at -2)	0.3 (Start: Pole at -3)	0 (End: Pole at -1)	20 (End: Pole at -2)	30 (End: Zero at -3)
Pole at -1	-45	-45	-45	0		
Pole at -2		-45	-45	-45	0	
Zero at -3			45	45	45	0
Total slope (deg/dec)	-45	-90	-45	0	45	0

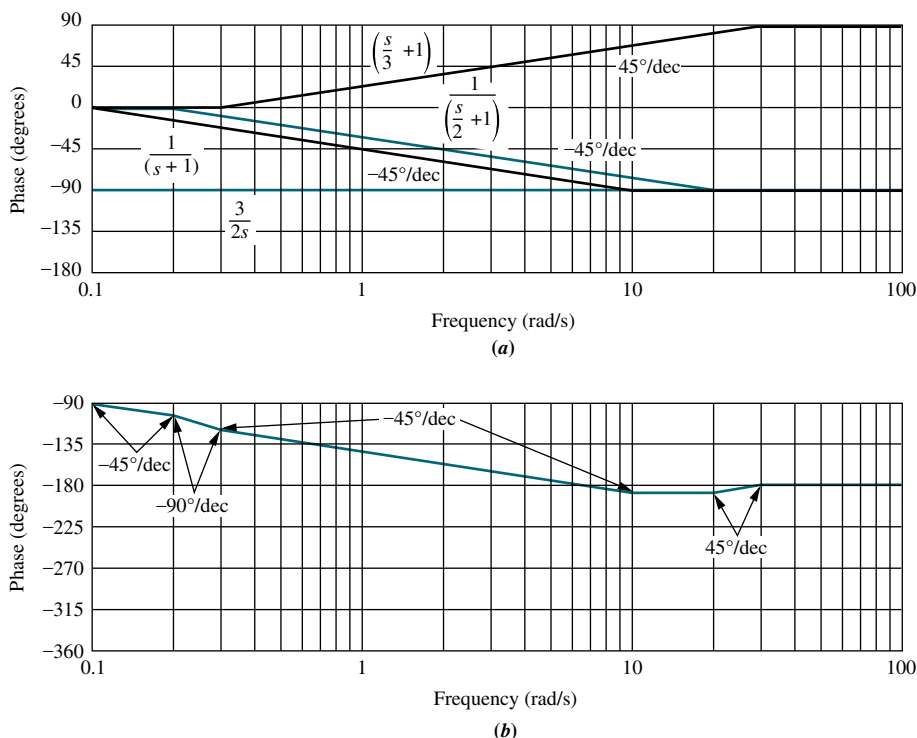


FIGURE 10.12 Bode phase plot for Example 10.2:
a. components;
b. composite

each of the poles and zeros. For example, reading across for the pole at -2 , we see that the -45° slope starts at a frequency of 0.2 and ends at 20. Filling in the rows for each pole and then summing the columns yields the slope portrait of the resulting phase plot. Looking at the row marked *Total slope*, we see that the phase plot will have a slope of $-45^\circ/\text{decade}$ from a frequency of 0.1 to 0.2. The slope will then increase to $-90^\circ/\text{decade}$ from 0.2 to 0.3. The slope will return to $-45^\circ/\text{decade}$ from 0.3 to 10 rad/s. A slope of 0 ensues from 10 to 20 rad/s, followed by a slope of $+45^\circ/\text{decade}$ from 20 to 30 rad/s. Finally, from 30 rad/s to infinity, the slope is $0^\circ/\text{decade}$.

The resulting component and composite phase plots are shown in Figure 10.12. Since the pole at the origin yields a constant -90° phase shift, the plot begins at -90° and follows the slope portrait just described.

Bode Plots for $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

Now that we have covered Bode plots for first-order systems, we turn to the Bode log-magnitude and phase plots for second-order polynomials in s . The second-order polynomial is of the form

$$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = \omega_n^2 \left(\frac{s^2}{\omega_n^2} + 2\zeta \frac{s}{\omega_n} + 1 \right) \quad (10.26)$$

Unlike the first-order frequency response approximation, the difference between the asymptotic approximation and the actual frequency response can be great for some values of ζ . A correction to the Bode diagrams can be made to improve the accuracy. We first derive the asymptotic approximation and then show the difference between the asymptotic approximation and the actual frequency response curves.

At low frequencies, Eq. (10.26) becomes

$$G(s) \approx \omega_n^2 = \omega_n^2 \angle 0^\circ \quad (10.27)$$

The magnitude, M , in dB at low frequencies therefore is

$$20 \log M = 20 \log |G(j\omega)| = 20 \log \omega_n^2 \quad (10.28)$$

At high frequencies,

$$G(s) \approx s^2 \quad (10.29)$$

or

$$G(j\omega) \approx -\omega^2 = \omega^2 \angle 180^\circ \quad (10.30)$$

The log-magnitude is

$$20 \log M = 20 \log |G(j\omega)| = 20 \log \omega^2 = 40 \log \omega \quad (10.31)$$

Equation (10.31) is a straight line with twice the slope of a first-order term (Eq. (10.20)). Its slope is 12 dB/octave, or 40 dB/decade.

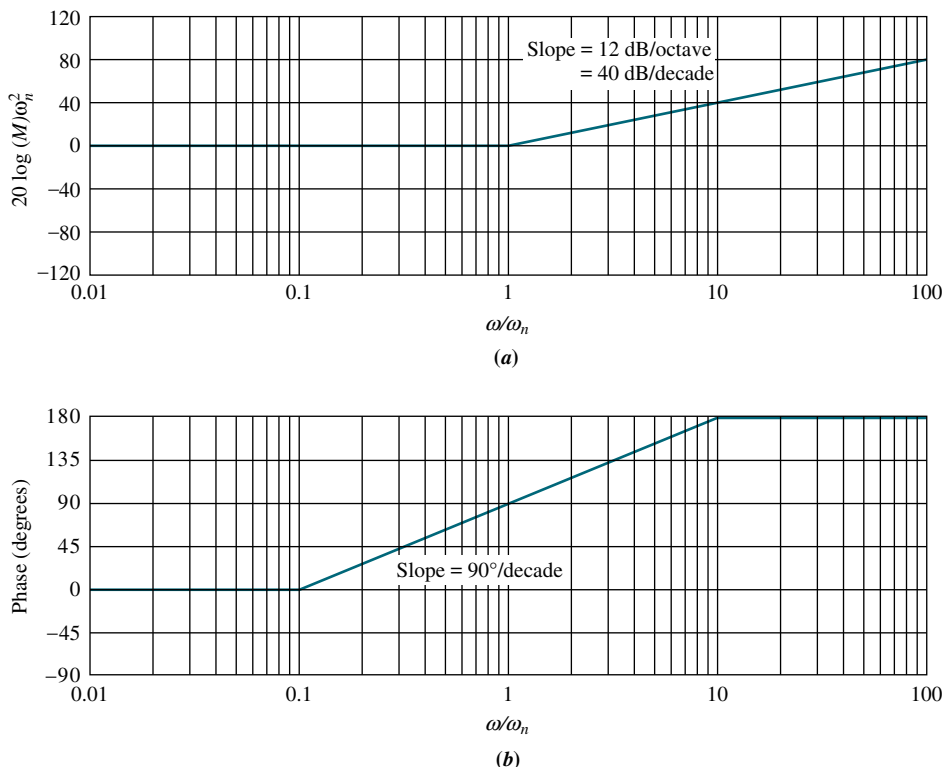


FIGURE 10.13

Bode asymptotes for normalized and scaled $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$:
a. magnitude; **b.** phase

The low-frequency asymptote (Eq. (10.27)) and the high-frequency asymptote (Eq. (10.31)) are equal when $\omega = \omega_n$. Thus, ω_n is the break frequency for the second-order polynomial.

For convenience in representing systems with different ω_n , we normalize and scale our findings before drawing the asymptotes. Using the normalized and scaled term of Eq. (10.26), we normalize the magnitude, dividing by ω_n^2 , and scale the frequency, dividing by ω_n . Thus, we plot $G(s_1)/\omega_n^2 = s_1^2 + 2\zeta s_1 + 1$, where $s_1 = s/\omega_n$. $G(s_1)$ has a low-frequency asymptote at 0 dB and a break frequency of 1 rad/s. Figure 10.13(a) shows the asymptotes for the normalized and scaled magnitude plot.

We now draw the phase plot. It is 0° at low frequencies (Eq. (10.27)) and 180° at high frequencies (Eq. (10.30)). To find the phase at the natural frequency, first evaluate $G(j\omega)$:

$$G(j\omega) = s^2 + 2\zeta\omega_n s + \omega_n^2 \Big|_{s \rightarrow j\omega} = (\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega \quad (10.32)$$

Then find the function value at the natural frequency by substituting $\omega = \omega_n$. Since the result is $j2\zeta\omega_n^2$, the phase at the natural frequency is $+90^\circ$. Figure 10.13(b) shows the phase plotted with frequency scaled by ω_n . The phase plot increases at a rate of $90^\circ/\text{decade}$ from 0.1 to 10 and passes through 90° at 1.

Corrections to Second-Order Bode Plots

Let us now examine the error between the actual response and the asymptotic approximation of the second-order polynomial. Whereas the first-order polynomial has a disparity of no more than 3.01 dB magnitude and 5.71° phase, the second-order function may have a greater disparity, which depends upon the value of ζ .

From Eq. (10.32), the actual magnitude and phase for $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$ are, respectively,

$$M = \sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} \tag{10.33}$$

$$\text{Phase} = \tan^{-1} \frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} \tag{10.34}$$

These relationships are tabulated in Table 10.4 for a range of values of ζ and plotted in Figures 10.14 and 10.15 along with the asymptotic approximations for normalized

TABLE 10.4 Data for normalized and scaled log-magnitude and phase plots for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$. $\text{Mag} = 20 \log(M/\omega_n^2)$

Freq. $\frac{\omega}{\omega_n}$	Phase (deg)		Phase (deg)		Phase (deg)	
	$\zeta = 0.1$	$\zeta = 0.1$	$\zeta = 0.2$	$\zeta = 0.2$	$\zeta = 0.3$	$\zeta = 0.3$
0.10	-0.09	1.16	-0.08	2.31	-0.07	3.47
0.20	-0.35	2.39	-0.32	4.76	-0.29	7.13
0.30	-0.80	3.77	-0.74	7.51	-0.65	11.19
0.40	-1.48	5.44	-1.36	10.78	-1.17	15.95
0.50	-2.42	7.59	-2.20	14.93	-1.85	21.80
0.60	-3.73	10.62	-3.30	20.56	-2.68	29.36
0.70	-5.53	15.35	-4.70	28.77	-3.60	39.47
0.80	-8.09	23.96	-6.35	41.63	-4.44	53.13
0.90	-11.64	43.45	-7.81	62.18	-4.85	70.62
1.00	-13.98	90.00	-7.96	90.00	-4.44	90.00
1.10	-10.34	133.67	-6.24	115.51	-3.19	107.65
1.20	-6.00	151.39	-3.73	132.51	-1.48	121.43
1.30	-2.65	159.35	-1.27	143.00	0.35	131.50
1.40	0.00	163.74	0.92	149.74	2.11	138.81
1.50	2.18	166.50	2.84	154.36	3.75	144.25
1.60	4.04	168.41	4.54	157.69	5.26	148.39
1.70	5.67	169.80	6.06	160.21	6.64	151.65
1.80	7.12	170.87	7.43	162.18	7.91	154.26
1.90	8.42	171.72	8.69	163.77	9.09	156.41
2.00	9.62	172.41	9.84	165.07	10.19	158.20
3.00	18.09	175.71	18.16	171.47	18.28	167.32
4.00	23.53	176.95	23.57	173.91	23.63	170.91
5.00	27.61	177.61	27.63	175.24	27.67	172.87
6.00	30.89	178.04	30.90	176.08	30.93	174.13
7.00	33.63	178.33	33.64	176.66	33.66	175.00
8.00	35.99	178.55	36.00	177.09	36.01	175.64
9.00	38.06	178.71	38.07	177.42	38.08	176.14
10.00	39.91	178.84	39.92	177.69	39.93	176.53

(table continues)

TABLE 10.4 Data for normalized and scaled log-magnitude and phase plots for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Mag = $20 \log(M/\omega_n^2)$
(Continued)

Freq. $\frac{\omega}{\omega_n}$	Mag (dB) $\zeta = 0.5$	Phase (deg) $\zeta = 0.5$	Mag (dB) $\zeta = 0.7$	Phase (deg) $\zeta = 0.7$	Mag (dB) $\zeta = 0.1$	Phase (deg) $\zeta = 0.1$
0.10	-0.04	5.77	0.00	8.05	0.09	11.42
0.20	-0.17	11.77	0.00	16.26	0.34	22.62
0.30	-0.37	18.25	0.02	24.78	0.75	33.40
0.40	-0.63	25.46	0.08	33.69	1.29	43.60
0.50	-0.90	33.69	0.22	43.03	1.94	53.13
0.60	-1.14	43.15	0.47	52.70	2.67	61.93
0.70	-1.25	53.92	0.87	62.51	3.46	69.98
0.80	-1.14	65.77	1.41	72.18	4.30	77.32
0.90	-0.73	78.08	2.11	81.42	5.15	83.97
1.00	0.00	90.00	2.92	90.00	6.02	90.00
1.10	0.98	100.81	3.83	97.77	6.89	95.45
1.20	2.13	110.14	4.79	104.68	7.75	100.39
1.30	3.36	117.96	5.78	110.76	8.60	104.86
1.40	4.60	124.44	6.78	116.10	9.43	108.92
1.50	5.81	129.81	7.76	120.76	10.24	112.62
1.60	6.98	134.27	8.72	124.85	11.03	115.99
1.70	8.10	138.03	9.66	128.45	11.80	119.07
1.80	9.17	141.22	10.56	131.63	12.55	121.89
1.90	10.18	143.95	11.43	134.46	13.27	124.48
2.00	11.14	146.31	12.26	136.97	13.98	126.87
3.00	18.63	159.44	19.12	152.30	20.00	143.13
4.00	23.82	165.07	24.09	159.53	24.61	151.93
5.00	27.79	168.23	27.96	163.74	28.30	157.38
6.00	31.01	170.27	31.12	166.50	31.36	161.08
7.00	33.72	171.70	33.80	168.46	33.98	163.74
8.00	36.06	172.76	36.12	169.92	36.26	165.75
9.00	38.12	173.58	38.17	171.05	38.28	167.32
10.00	39.96	174.23	40.00	171.95	40.09	168.58

magnitude and scaled frequency. In Figure 10.14, which is normalized to the square of the natural frequency, the normalized log-magnitude at the scaled natural frequency is $+20 \log 2\zeta$. The student should verify that the actual magnitude at the unscaled natural frequency is $+20 \log 2\zeta\omega_n^2$. Table 10.4 and Figures 10.14 and 10.15 can be used to improve accuracy when drawing Bode plots. For example, a magnitude correction of $+20 \log 2\zeta$ can be made at the natural, or break, frequency on the Bode asymptotic plot.

Bode Plots for $G(s) = 1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

Bode plots for $G(s) = 1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$ can be derived similarly to those for $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$. We find that the magnitude curve breaks at the natural frequency and decreases at a rate of -40 dB/decade. The phase plot is 0° at low

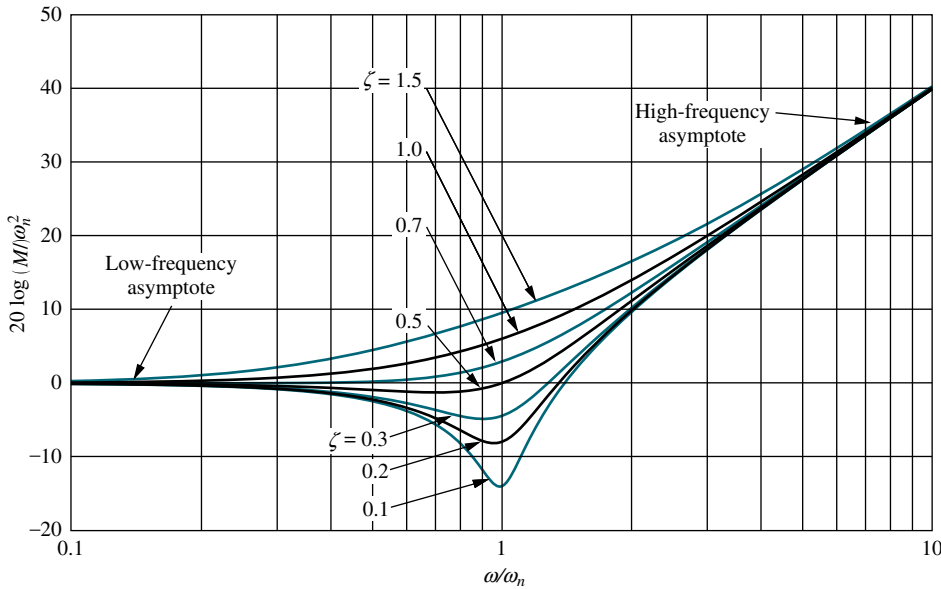


FIGURE 10.14 Normalized and scaled log-magnitude response for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$

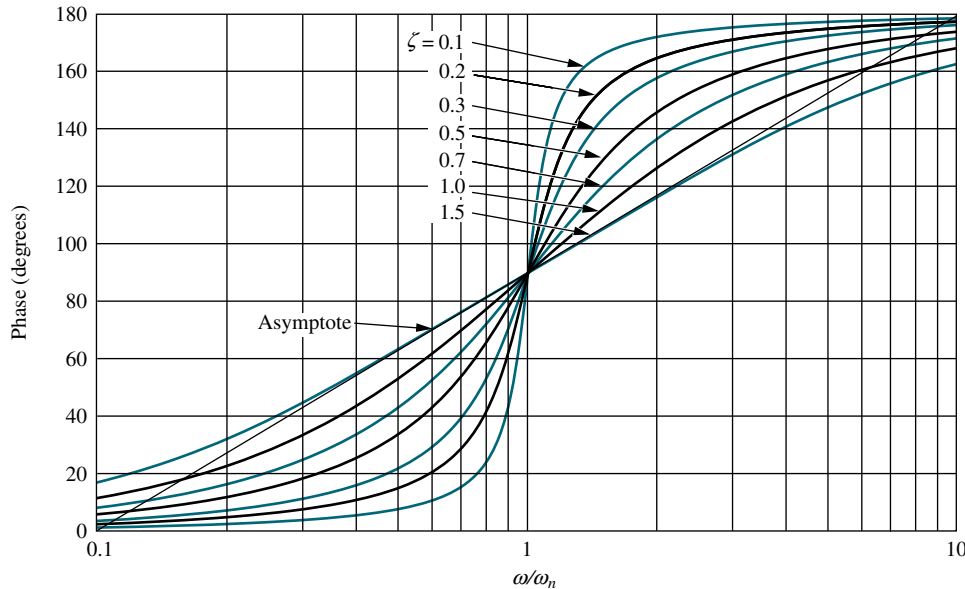


FIGURE 10.15 Scaled phase response for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$

frequencies. At $0.1\omega_n$ it begins a decrease of $-90^\circ/\text{decade}$ and continues until $\omega = 10\omega_n$, where it levels off at -180° .

The exact frequency response also follows the same derivation as that of $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$. The results are summarized in Table 10.5, as well as Figures 10.16 and 10.17. The exact magnitude is the reciprocal of Eq. (10.33), and the exact phase is the negative of Eq. (10.34). The normalized magnitude at the scaled natural frequency is $-20 \log 2\zeta$, which can be used as a correction at the break frequency on the Bode asymptotic plot.

TABLE 10.5 Data for normalized and scaled log-magnitude and phase plots for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Mag = $20 \log(M/\omega_n^2)$

Freq. $\frac{\omega}{\omega_n}$	Mag (dB) $\zeta = 0.1$	Phase (deg) $\zeta = 0.1$	Mag (dB) $\zeta = 0.2$	Phase (deg) $\zeta = 0.2$	Mag (dB) $\zeta = 0.3$	Phase (deg) $\zeta = 0.3$
0.10	0.09	-1.16	0.08	-2.31	0.07	-3.47
0.20	0.35	-2.39	0.32	-4.76	0.29	-7.13
0.30	0.80	-3.77	0.74	-7.51	0.65	-11.19
0.40	1.48	-5.44	1.36	-10.78	1.17	-15.95
0.50	2.42	-7.59	2.20	-14.93	1.85	-21.80
0.60	3.73	-10.62	3.30	-20.56	2.68	-29.36
0.70	5.53	-15.35	4.70	-28.77	3.60	-39.47
0.80	8.09	-23.96	6.35	-41.63	4.44	-53.13
0.90	11.64	-43.45	7.81	-62.18	4.85	-70.62
1.00	13.98	-90.00	7.96	-90.00	4.44	-90.00
1.10	10.34	-133.67	6.24	-115.51	3.19	-107.65
1.20	6.00	-151.39	3.73	-132.51	1.48	-121.43
1.30	2.65	-159.35	1.27	-143.00	-0.35	-131.50
1.40	0.00	-163.74	-0.92	-149.74	-2.11	-138.81
1.50	-2.18	-166.50	-2.84	-154.36	-3.75	-144.25
1.60	-4.04	-168.41	-4.54	-157.69	-5.26	-148.39
1.70	-5.67	-169.80	-6.06	-160.21	-6.64	-151.65
1.80	-7.12	-170.87	-7.43	-162.18	-7.91	-154.26
1.90	-8.42	-171.72	-8.69	-163.77	-9.09	-156.41
2.00	-9.62	-172.41	-9.84	-165.07	-10.19	-158.20
3.00	-18.09	-175.71	-18.16	-171.47	-18.28	-167.32
4.00	-23.53	-176.95	-23.57	-173.91	-23.63	-170.91
5.00	-27.61	-177.61	-27.63	-175.24	-27.67	-172.87
6.00	-30.89	-178.04	-30.90	-176.08	-30.93	-174.13
7.00	-33.63	-178.33	-33.64	-176.66	-33.66	-175.00
8.00	-35.99	-178.55	-36.00	-177.09	-36.01	-175.64
9.00	-38.06	-178.71	-38.07	-177.42	-38.08	-176.14
10.00	-39.91	-178.84	-39.92	-177.69	-39.93	-176.53

(table continues)

TABLE 10.5 Data for normalized and scaled log-magnitude and phase plots for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. $\text{Mag} = 20 \log(M/\omega_n^2)$
(Continued)

Freq. $\frac{\omega}{\omega_n}$	Mag (dB)		Phase (deg)		Mag (dB)		Phase (deg)	
	$\zeta = 0.5$	$\zeta = 0.5$	$\zeta = 0.7$	$\zeta = 0.7$	$\zeta = 0.1$	$\zeta = 0.1$	$\zeta = 0.1$	$\zeta = 0.1$
0.10	0.04	-5.77	0.00	-8.05	-0.09		-11.42	
0.20	0.17	-11.77	0.00	-16.26	-0.34		-22.62	
0.30	0.37	-18.25	-0.02	-24.78	-0.75		-33.40	
0.40	0.63	-25.46	-0.08	-33.69	-1.29		-43.60	
0.50	0.90	-33.69	-0.22	-43.03	-1.94		-53.13	
0.60	1.14	-43.15	-0.47	-52.70	-2.67		-61.93	
0.70	1.25	-53.92	-0.87	-62.51	-3.46		-69.98	
0.80	1.14	-65.77	-1.41	-72.18	-4.30		-77.32	
0.90	0.73	-78.08	-2.11	-81.42	-5.15		-83.97	
1.00	0.00	-90.00	-2.92	-90.00	-6.02		-90.00	
1.10	-0.98	-100.81	-3.93	-97.77	-6.89		-95.45	
1.20	-2.13	-110.14	-4.79	-104.68	-7.75		-100.39	
1.30	-3.36	-117.96	-5.78	-110.76	-8.60		-104.86	
1.40	-4.60	-124.44	-6.78	-116.10	-9.43		-108.92	
1.50	-5.81	-129.81	-7.76	-120.76	-10.24		-112.62	
1.60	-6.98	-134.27	-8.72	-124.85	-11.03		-115.99	
1.70	-8.10	-138.03	-9.66	-128.45	-11.80		-119.07	
1.80	-9.17	-141.22	-10.56	-131.63	-12.55		-121.89	
1.90	-10.18	-143.95	-11.43	-134.46	-13.27		-124.48	
2.00	-11.14	-146.31	-12.26	-136.97	-13.98		-126.87	
3.00	-18.63	-159.44	-19.12	-152.30	-20.00		-143.13	
4.00	-23.82	-165.07	-24.09	-159.53	-24.61		-151.93	
5.00	-27.79	-168.23	-27.96	-163.74	-28.30		-157.38	
6.00	-31.01	-170.27	-31.12	-166.50	-31.36		-161.08	
7.00	-33.72	-171.70	-33.80	-168.46	-33.98		-163.74	
8.00	-36.06	-172.76	-36.12	-169.92	-36.26		-165.75	
9.00	-38.12	-173.58	-38.17	-171.05	-38.28		-167.32	
10.00	-39.96	-174.23	-40.00	-171.95	-40.09		-168.58	

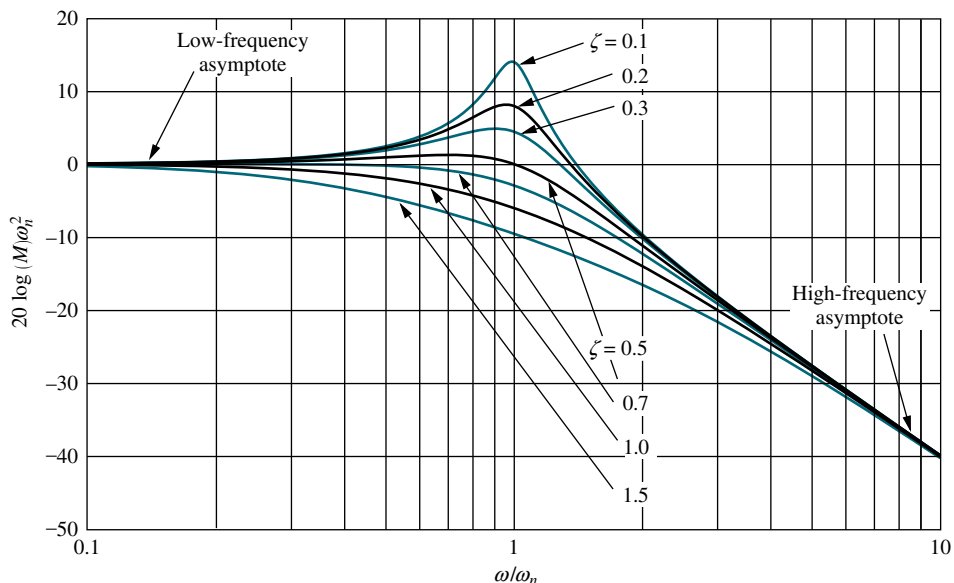


FIGURE 10.16 Normalized and scaled log-magnitude response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

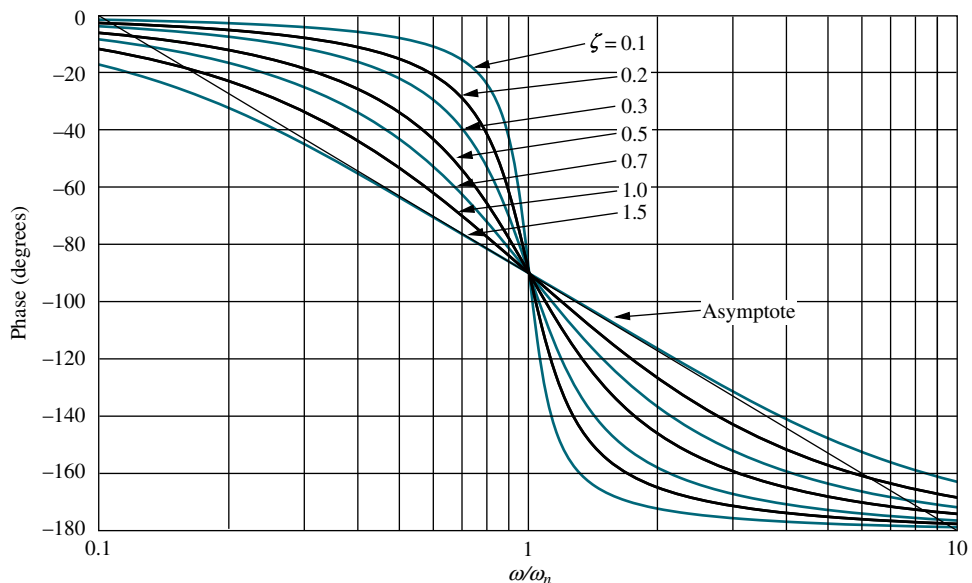


FIGURE 10.17 Scaled phase response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

Let us now look at an example of drawing Bode plots for transfer functions that contain second-order factors.

Example 10.3

Bode Plots for Ratio of First- and Second-Order Factors

PROBLEM: Draw the Bode log-magnitude and phase plots of $G(s)$ for the unity feedback system shown in Figure 10.10, where $G(s) = (s + 3)/[(s + 2)(s^2 + 2s + 25)]$.

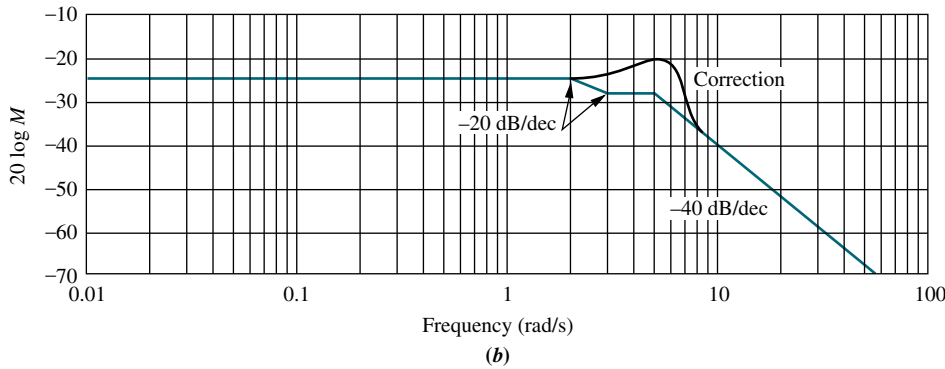
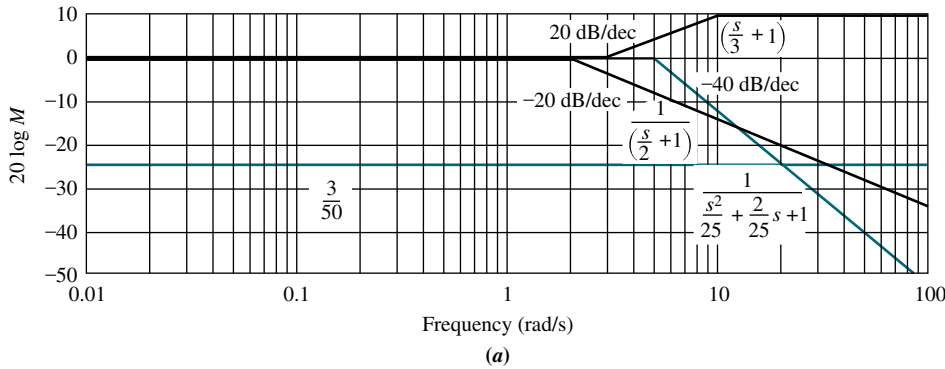


FIGURE 10.18 Bode magnitude plot for $G(s) = (s + 3)/[(s + 2)(s^2 + 2s + 25)]$: **a.** components; **b.** composite

SOLUTION: We first convert $G(s)$ to show the normalized components that have unity low-frequency gain. The second-order term is normalized by factoring out ω_n^2 , forming

$$\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1 \tag{10.35}$$

Thus,

$$G(s) = \frac{3}{(2)(25)} \frac{\left(\frac{s}{3} + 1\right)}{\left(\frac{s}{2} + 1\right)\left(\frac{s^2}{25} + \frac{2}{25}s + 1\right)} = \frac{3}{50} \frac{\left(\frac{s}{3} + 1\right)}{\left(\frac{s}{2} + 1\right)\left(\frac{s^2}{25} + \frac{2}{25}s + 1\right)} \tag{10.36}$$

The Bode log-magnitude diagram is shown in Figure 10.18(b) and is the sum of the individual first- and second-order terms of $G(s)$ shown in Figure 10.18(a). We solve this problem by adding the slopes of these component parts, beginning and ending at the appropriate frequencies. The results are summarized in Table 10.6, which can be used to obtain the slopes. The low-frequency value for $G(s)$, found by

TABLE 10.6 Magnitude diagram slopes for Example 10.3

Description	Frequency (rad/s)			
	0.01 (Start: Plot)	2 (Start: Pole at -2)	3 (Start: Zero at -3)	5 (Start: $\omega_n = 5$)
Pole at -2	0	-20	-20	-20
Zero at -3	0	0	20	20
$\omega_n = 5$	0	0	0	-40
Total slope (dB/dec)	0	-20	0	-40

letting $s = 0$, is $3/50$, or -24.44 dB. The Bode magnitude plot starts out at this value and continues until the first break frequency at 2 rad/s. Here the pole at -2 yields a -20 dB/decade slope downward until the next break at 3 rad/s. The zero at -3 causes an upward slope of $+20$ dB/decade, which, when added to the previous -20 dB/decade curve, gives a net slope of 0. At a frequency of 5 rad/s, the second-order term initiates a -40 dB/decade downward slope, which continues to infinity.

The correction to the log-magnitude curve due to the underdamped second-order term can be found by plotting a point $-20 \log 2\zeta$ above the asymptotes at the natural frequency. Since $\zeta = 0.2$ for the second-order term in the denominator of $G(s)$, the correction is 7.96 dB. Points close to the natural frequency can be corrected by taking the values from the curves of Figure 10.16.

TABLE 10.7 Phase diagram slopes for Example 10.3

Description	Frequency (rad/s)					
	0.2 (Start: Pole at -2)	0.3 (Start: Zero at -3)	0.5 (Start: ω_n at -5)	20 (End: Pole at -2)	30 (End: Zero at -3)	50 (End: $\omega_n = 5$)
Pole at -2	-45	-45	-45	0		
Zero at -3		45	45	45	0	
$\omega_n = 5$			-90	-90	-90	0
Total slope (dB/dec)	-45	0	-90	-45	-90	0

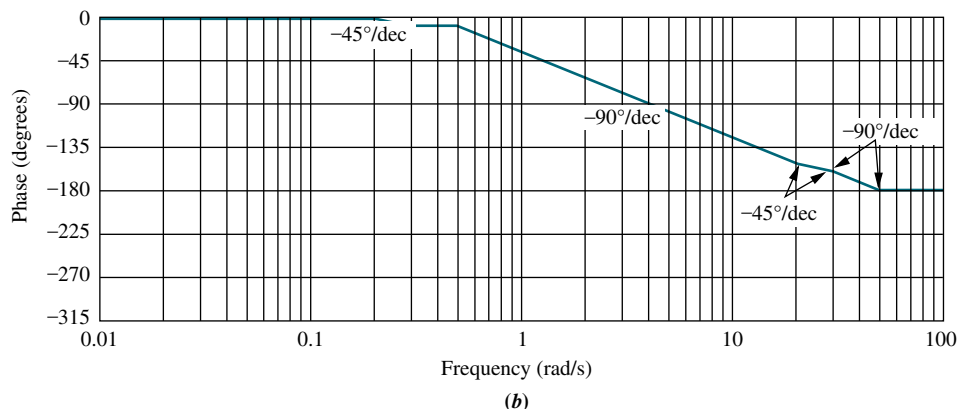
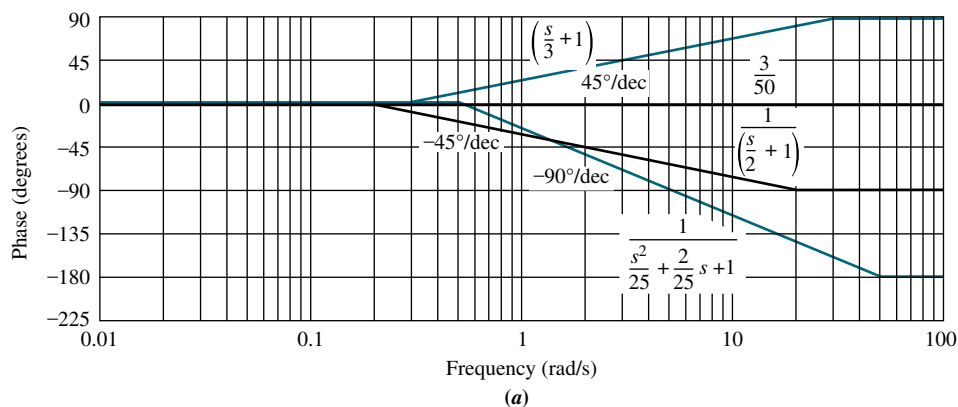


FIGURE 10.19 Bode phase plot for $G(s) = (s + 3)/[(s + 2)(s^2 + 2s + 25)]$:
a. components;
b. composite

We now turn to the phase plot. Table 10.7 is formed to determine the progression of slopes on the phase diagram. The first-order pole at -2 yields a phase angle that starts at 0° and ends at -90° via a $-45^\circ/\text{decade}$ slope starting a decade below its break frequency and ending a decade above its break frequency. The first-order zero yields a phase angle that starts at 0° and ends at $+90^\circ$ via a $+45^\circ/\text{decade}$ slope starting a decade below its break frequency and ending a decade above its break frequency. The second-order poles yield a phase angle that starts at 0° and ends at -180° via a $-90^\circ/\text{decade}$ slope starting a decade below their natural frequency ($\omega_n = 5$) and ending a decade above their natural frequency. The slopes, shown in Figure 10.19(a), are summed over each frequency range, and the final Bode phase plot is shown in Figure 10.19(b).

Students who are using MATLAB should now run `ch10p1` in Appendix B. You will learn how to use MATLAB to make Bode plots and list the points on the plots. This exercise solves Example 10.3 using MATLAB.

MATLAB

ML

Skill-Assessment Exercise 10.2

PROBLEM: Draw the Bode log-magnitude and phase plots for the system shown in Figure 10.10, where

$$G(s) = \frac{(s + 20)}{(s + 1)(s + 7)(s + 50)}$$

WileyPLUS
WPCS
 Control Solutions

ANSWER: The complete solution is at www.wiley.com/college/nise.

TryIt 10.1

Use MATLAB, the Control System Toolbox, and the following statements to obtain the Bode plots for the system of Skill-Assessment Exercise 10.2

```
G=zpk([-20],[-1,-7,...
-50],1)
bode(G);grid on
```

After the Bode plots appear, click on the curve and drag to read the coordinates.

In this section, we learned how to construct Bode log-magnitude and Bode phase plots. The Bode plots are separate magnitude and phase frequency response curves for a system, $G(s)$. In the next section, we develop the Nyquist criterion for stability, which makes use of the frequency response of a system. The Bode plots can then be used to determine the stability of a system.

10.3 Introduction to the Nyquist Criterion

The Nyquist criterion relates the stability of a closed-loop system to the open-loop frequency response and open-loop pole location. Thus, knowledge of the open-loop system's frequency response yields information about the stability of the closed-loop system. This concept is similar to the root locus, where we began with information about the open-loop system, its poles and zeros, and developed transient and stability information about the closed-loop system.