7.4 CANONICAL FORM OF A FEEDBACK CONTROL SYSTEM

The two blocks in the forward path of the feedback system of Fig. 7-2 may be combined. Letting $G \equiv G_1 G_2$, the resulting configuration is called the **canonical form** of a feedback control system. G and H are not necessarily unique for a particular system.

The following definitions refer to Fig. 7-5.

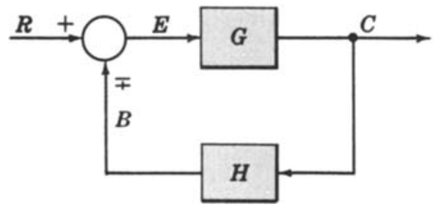


Fig. 7-5

Definition 7.1: $G \equiv$ direct transfer function $\equiv$ forward transfer function

Definition 7.2: $H \equiv$ feedback transfer function

Definition 7.3: $GH \equiv$ loop transfer function $\equiv$ open-loop transfer function

Definition 7.4: $C/R \equiv$ closed-loop transfer function $\equiv$ control ratio

Definition 7.5: $E/R \equiv$ actuating signal ratio $\equiv$ error ratio

Definition 7.6: $B/R \equiv$ primary feedback ratio

In the following equations, the $-$ sign refers to a *positive* feedback system, and the $+$ sign refers to a *negative* feedback system:

$$\frac{C}{R} = \frac{G}{1 \pm GH} \tag{7.3}$$

$$\frac{E}{R} = \frac{1}{1 \pm GH} \tag{7.4}$$

$$\frac{B}{R} = \frac{GH}{1 \pm GH} \tag{7.5}$$

The denominator of C/R determines the *characteristic equation* of the system, which is usually determined from $1 \pm GH = 0$ or, equivalently,

$$D_{GH} \pm N_{GH} = 0 \tag{7.6}$$

where D_{GH} is the denominator and N_{GH} is the numerator of GH, unless a pole of G cancels a zero of H (see Problem 7.9). Relations (*7.1*) through (*7.6*) are valid for both continuous (s-domain) and discrete (z-domain) systems.

7.5 BLOCK DIAGRAM TRANSFORMATION THEOREMS

Block diagrams of complicated control systems may be simplified using easily derivable transformations. The first important transformation, combining blocks in cascade, has already been presented in Section 7.3. It is repeated for completeness in the chart illustrating the transformation theorems (Fig. 7-6). The letter P is used to represent any transfer function, and W, X, Y, Z denote any transformed signals.

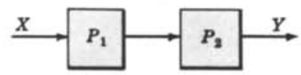
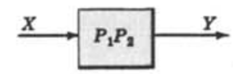
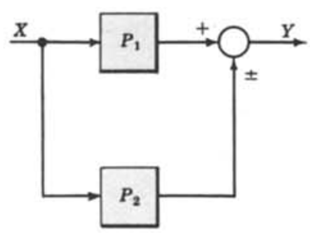
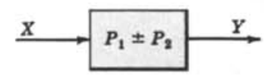
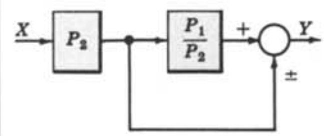
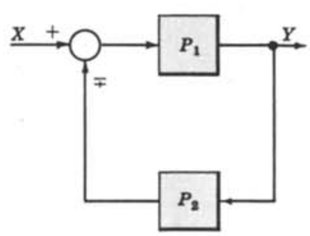
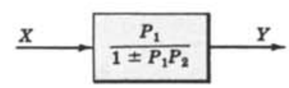
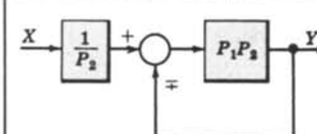
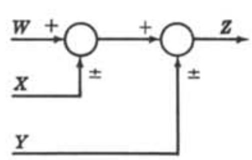
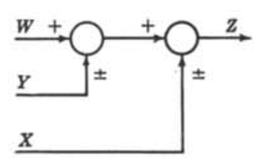
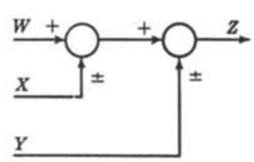
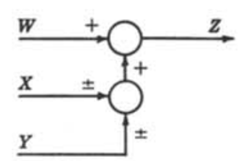
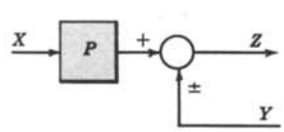
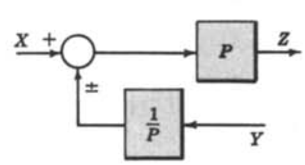
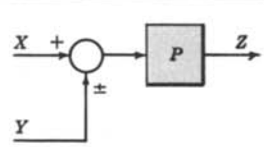
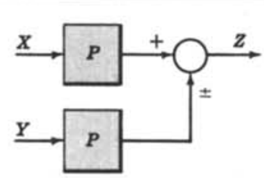
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1X \pm P_2X$		
3	Removing a Block from a Forward Path	$Y = P_1X \pm P_2X$		
4	Eliminating a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
5	Removing a Block from a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
6a	Rearranging Summing Points	$Z = W \pm X \pm Y$		
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = PX \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P[X \pm Y]$		

Fig. 7-6

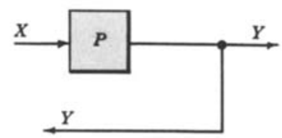
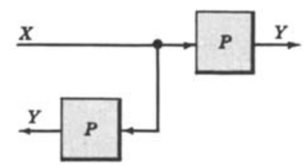
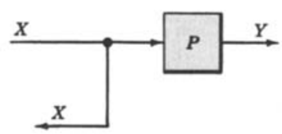
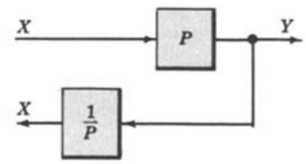
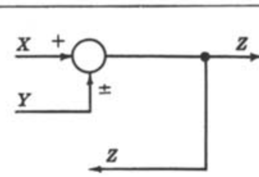
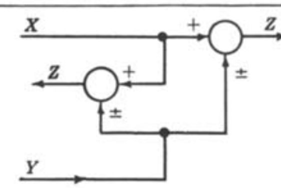
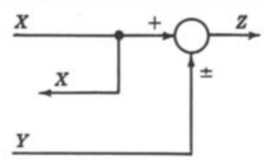
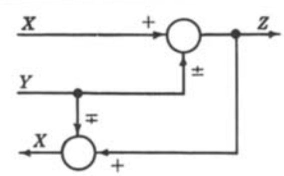
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

Fig. 7-6 *Continued*

7.6 UNITY FEEDBACK SYSTEMS

Definition 7.7: A **unity feedback system** is one in which the primary feedback b is identically equal to the controlled output c.

EXAMPLE 7.6. $H = 1$ for a linear, unity feedback system (Fig. 7-7).

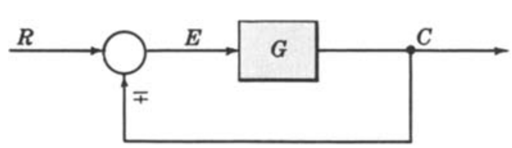


Fig. 7-7

Any feedback system with only linear time-invariant elements can be put into the form of a unity feedback system by using Transformation 5.

EXAMPLE 7.7.

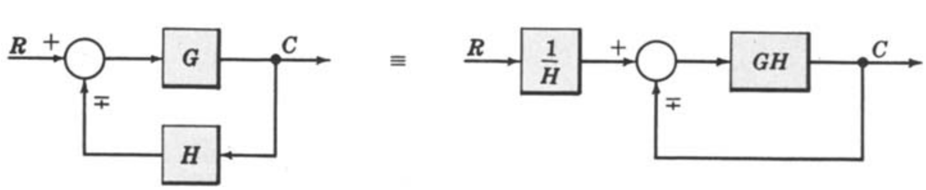


Fig. 7-8

The characteristic equation for the unity feedback system, determined from $1 \pm G = 0$, is

$$D_G \pm N_G = 0 \tag{7.7}$$

where D_G is the denominator and N_G the numerator of G.

7.7 SUPERPOSITION OF MULTIPLE INPUTS

Sometimes it is necessary to evaluate system performance when several inputs are simultaneously applied at different points of the system.

When multiple inputs are present in a *linear* system, each is treated independently of the others. The output due to all stimuli acting together is found in the following manner. We assume zero initial conditions, as we seek the system response only to inputs.

Step 1: Set all inputs except one equal to zero.

Step 2: Transform the block diagram to canonical form, using the transformations of Section 7.5.

Step 3: Calculate the response due to the chosen input acting alone.

Step 4: Repeat Steps 1 to 3 for each of the remaining inputs.

Step 5: Algebraically add all of the responses (outputs) determined in Steps 1 to 4. This sum is the total output of the system with all inputs acting simultaneously.

We reemphasize here that the above superposition process is dependent on the system being linear.

EXAMPLE 7.8. We determine the output C due to inputs U and R for Fig. 7-9.

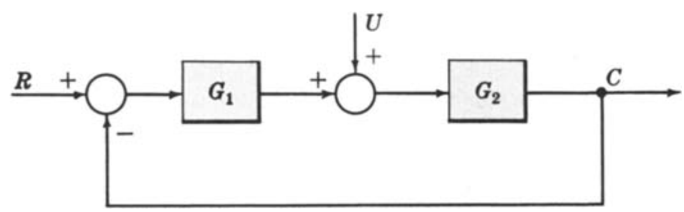
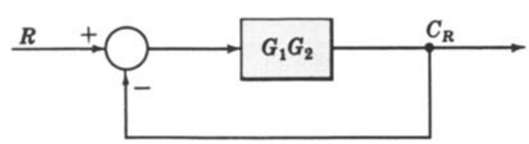
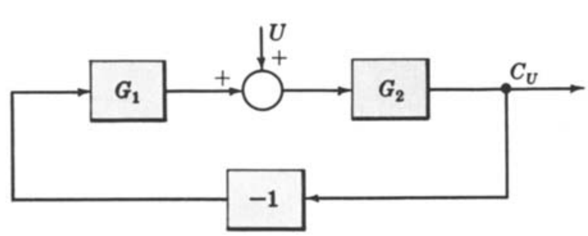


Fig. 7-9

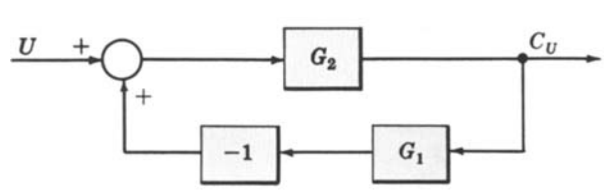
Step 1: Put $U \equiv 0$.
Step 2: The system reduces to



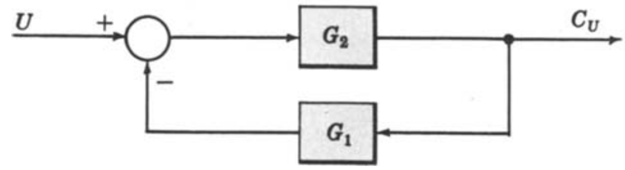
Step 3: By Equation (7.3), the output C_R due to input R is $C_R = [G_1G_2/(1 + G_1G_2)]R$.
Step 4a: Put $R = 0$.
Step 4b: Put -1 into a block, representing the negative feedback effect:



Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



Step 4c: By Equation (7.3), the output C_U due to input U is $C_U = [G_2/(1 + G_1G_2)]U$.
Step 5: The total output is

$$C = C_R + C_U = \left[\frac{G_1G_2}{1 + G_1G_2}\right]R + \left[\frac{G_2}{1 + G_1G_2}\right]U = \left[\frac{G_2}{1 + G_1G_2}\right][G_1R + U]$$

7.8 REDUCTION OF COMPLICATED BLOCK DIAGRAMS

The block diagram of a practical feedback control system is often quite complicated. It may include several feedback or feedforward loops, and multiple inputs. By means of systematic block diagram reduction, every multiple loop linear feedback system may be reduced to canonical form. The techniques developed in the preceding paragraphs provide the necessary tools.

The following general steps may be used as a basic approach in the reduction of complicated block diagrams. Each step refers to specific transformations listed in Fig. 7-6.

Step 1: Combine all cascade blocks using Transformation 1.

Step 2: Combine all parallel blocks using Transformation 2.

Step 3: Eliminate all minor feedback loops using Transformation 4.

Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12.

Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.

Step 6: Repeat Steps 1 to 5 for each input, as required.

Transformations 3, 5, 6, 8, 9, and 11 are sometimes useful, and experience with the reduction technique will determine their application.

EXAMPLE 7.9. Let us reduce the block diagram (Fig. 7-10) to canonical form.

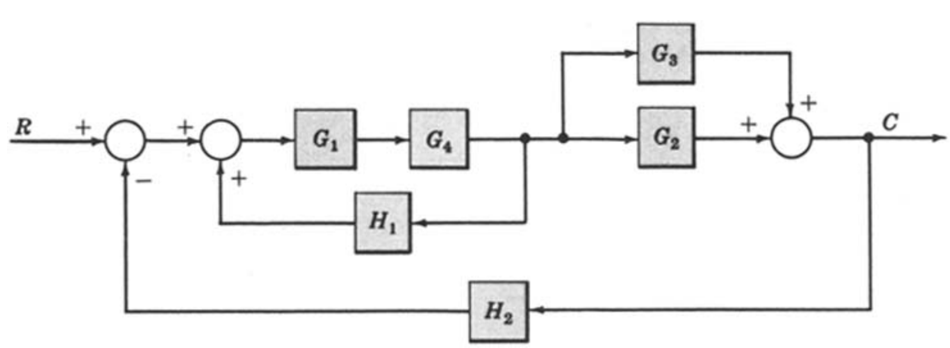


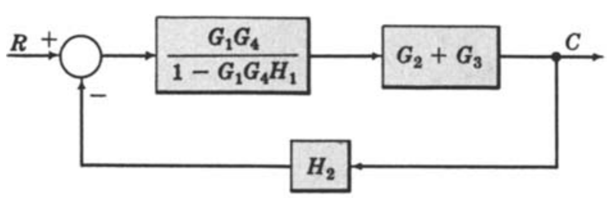
Fig. 7-10

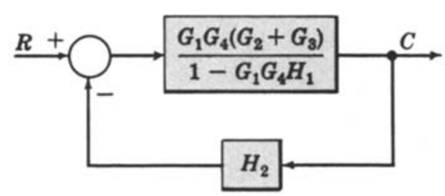
Step 1:

Step 2:

Step 3:

Step 4: Does not apply.
Step 5:





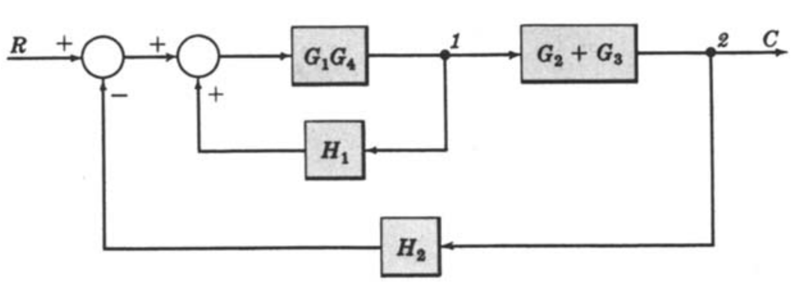
Step 6: Does not apply.

An occasional requirement of block diagram reduction is the isolation of a particular block in a feedback or feedforward loop. This may be desirable to more easily examine the effect of a particular block on the overall system.

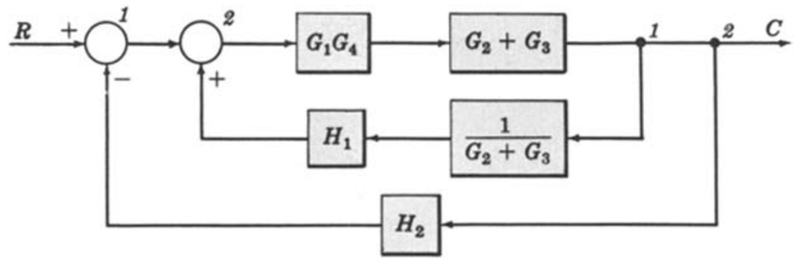
Isolation of a block generally may be accomplished by applying the same reduction steps to the system, but usually in a different order. Also, the block to be isolated cannot be combined with any others.

Rearranging Summing Points (Transformation 6) and Transformations 8, 9, and 11 are especially useful for isolating blocks.

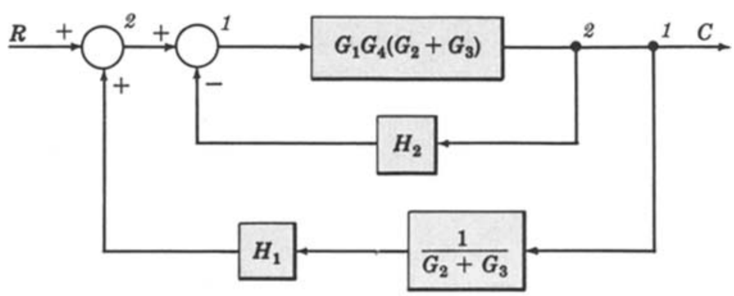
EXAMPLE 7.10. Let us reduce the block diagram of Example 7.9, isolating block H_1.
Steps 1 and 2:



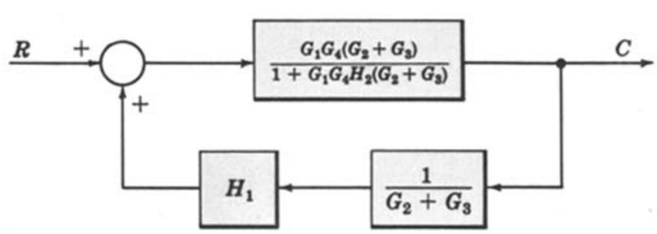
We do not apply Step 3 at this time, but go directly to Step 4, moving takeoff point *1* beyond block $G_2 + G_3$:



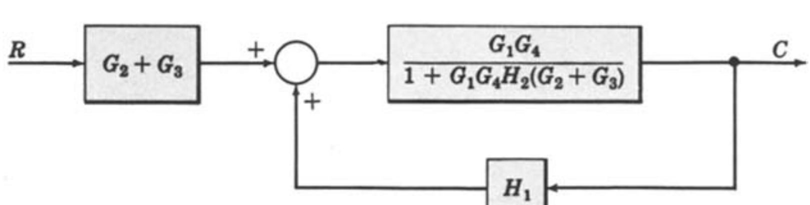
We may now rearrange summing points *1* and *2* and combine the cascade blocks in the forward loop using Transformation 6, then Transformation 1:



Step 3:



Finally, we apply Transformation 5 to remove $1/(G_2 + G_3)$ from the feedback loop:



Note that the same result could have been obtained after applying Step 2 by moving takeoff point *2 ahead* of $G_2 + G_3$, instead of takeoff point *1 beyond* $G_2 + G_3$. Block $G_2 + G_3$ has the same effect on the control ratio C/R whether it directly follows R or directly precedes C.