<table>
<thead>
<tr>
<th>Transformation</th>
<th>Equation</th>
<th>Block Diagram</th>
<th>Equivalent Block Diagram</th>
</tr>
</thead>
</table>
| 1 Combining Blocks in Cascade          | $Y = (P_1P_2)X$ | $\begin{align*}X & \xrightarrow{P_1} P_4 \xrightarrow{Y} \\
& \end{align*}$ | $\begin{align*}X & \xrightarrow{P_1} \xrightarrow{P_3} Y \\
& \end{align*}$ |
| 2 Combining Blocks in Parallel; or     | $Y = P_1X \pm P_2X$ | $\begin{align*}X & \xrightarrow{P_1} \xrightarrow{P_2} Y \\
& \pm \end{align*}$ | $\begin{align*}X & \xrightarrow{P_1 \pm P_3} Y \\
& \pm \end{align*}$ |
| Eliminating a Forward Loop             | $Y = P_1X \pm P_2X$ | $\begin{align*}X & \xrightarrow{P_3} Y \\
& \pm \end{align*}$ | $\begin{align*}X & \xrightarrow{P_2} \xrightarrow{P_1 \pm P_3} Y \\
& \pm \end{align*}$ |
| Removing a Block from a Feedback Loop  | $Y = P_1(X \mp P_2Y)$ | $\begin{align*}X & \xrightarrow{P_1} \xrightarrow{P_2} Y \\
& \mp \end{align*}$ | $\begin{align*}X & \xrightarrow{1 \mp P_1P_2} Y \\
& \mp \end{align*}$ |
| 5 Removing a Block from a Feedback     | $Y = P_1(X \mp P_2Y)$ | $\begin{align*}X & \xrightarrow{1 \mp P_1P_2} \xrightarrow{P_2P_3} Y \\
& \mp \end{align*}$ | $\begin{align*}X & \xrightarrow{P_2} \xrightarrow{P_1 \pm P_3} Y \\
& \mp \end{align*}$ |
| 6a Rearranging Summing Points          | $Z = W \mp X \pm Y$ | $\begin{align*}W & \xrightarrow{X} \xrightarrow{Z} Y \\
& \mp \end{align*}$ | $\begin{align*}W & \xrightarrow{X} \xrightarrow{Z} Y \\
& \mp \end{align*}$ |
| 6b Rearranging Summing Points          | $Z = W \mp X \pm Y$ | $\begin{align*}W & \xrightarrow{X} \xrightarrow{Z} Y \\
& \mp \end{align*}$ | $\begin{align*}W & \xrightarrow{X} \xrightarrow{Z} Y \\
& \mp \end{align*}$ |
| 7 Moving a Summing Point Ahead of a    | $Z = PX \mp Y$ | $\begin{align*}X & \xrightarrow{P} \xrightarrow{Z} Y \\
& \mp \end{align*}$ | $\begin{align*}X & \xrightarrow{P} \xrightarrow{1 \mp P} Y \\
& \mp \end{align*}$ |
| Block                                   |              |               |                          |
| 8 Moving a Summing Point Beyond a Block | $Z = P[X \mp Y]$ | $\begin{align*}X & \xrightarrow{P} \xrightarrow{Z} Y \\
& \mp \end{align*}$ | $\begin{align*}X & \xrightarrow{P} \xrightarrow{Z} Y \\
& \mp \end{align*}$ |

Fig. 7-6
7.6 UNITY FEEDBACK SYSTEMS

**Definition 7.7:** A unity feedback system is one in which the primary feedback \( b \) is identically equal to the controlled output \( c \).

**EXAMPLE 7.6.** \( H = 1 \) for a linear, unity feedback system (Fig. 7-7).

Any feedback system with only linear time-invariant elements can be put into the form of a unity feedback system by using Transformation 5.

**EXAMPLE 7.7.**
The characteristic equation for the unity feedback system, determined from $1 + G = 0$, is

$$D_G \pm N_G = 0 \quad (7.7)$$

where $D_G$ is the denominator and $N_G$ the numerator of $G$.

7.7 SUPERPOSITION OF MULTIPLE INPUTS

Sometimes it is necessary to evaluate system performance when several inputs are simultaneously applied at different points of the system.

When multiple inputs are present in a linear system, each is treated independently of the others. The output due to all stimuli acting together is found in the following manner. We assume zero initial conditions, as we seek the system response only to inputs.

**Step 1:** Set all inputs except one equal to zero.

**Step 2:** Transform the block diagram to canonical form, using the transformations of Section 7.5.

**Step 3:** Calculate the response due to the chosen input acting alone.

**Step 4:** Repeat Steps 1 to 3 for each of the remaining inputs.

**Step 5:** Algebraically add all of the responses (outputs) determined in Steps 1 to 4. This sum is the total output of the system with all inputs acting simultaneously.

We reemphasize here that the above superposition process is dependent on the system being linear.

**EXAMPLE 7.8.** We determine the output $C$ due to inputs $U$ and $R$ for Fig. 7-9.

**Step 1:** Put $U = 0$.

**Step 2:** The system reduces to

**Step 3:** By Equation (7.3), the output $C_R$ due to input $R$ is $C_R = \left[G_1G_2/(1 + G_1G_2)\right]R$.

**Step 4a:** Put $R = 0$.

**Step 4b:** Put $-1$ into a block, representing the negative feedback effect:

Rearrange the block diagram:
Let the \(-1\) block be absorbed into the summing point:

\[
\begin{align*}
U & \rightarrow G_2 \rightarrow + \rightarrow C_U \\
& \rightarrow - \rightarrow G_1 \rightarrow + \rightarrow \text{Summing Point}
\end{align*}
\]

**Step 4c:** By Equation (7.3), the output \(C_U\) due to input \(U\) is \(C_U = \frac{G_2}{1 + G_1 G_2} U\).

**Step 5:** The total output is

\[
C = C_R + C_U = \left[ \frac{G_1 G_2}{1 + G_1 G_2} \right] R + \left[ \frac{G_2}{1 + G_1 G_2} \right] U = \left[ \frac{G_2}{1 + G_1 G_2} \right] [G_1 R + U]
\]

### 7.8 REDUCTION OF COMPLICATED BLOCK DIAGRAMS

The block diagram of a practical feedback control system is often quite complicated. It may include several feedback or feedforward loops, and multiple inputs. By means of systematic block diagram reduction, every multiple loop linear feedback system may be reduced to canonical form. The techniques developed in the preceding paragraphs provide the necessary tools.

The following general steps may be used as a basic approach in the reduction of complicated block diagrams. Each step refers to specific transformations listed in Fig. 7-6.

- **Step 1:** Combine all cascade blocks using Transformation 1.
- **Step 2:** Combine all parallel blocks using Transformation 2.
- **Step 3:** Eliminate all minor feedback loops using Transformation 4.
- **Step 4:** Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12.
- **Step 5:** Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.
- **Step 6:** Repeat Steps 1 to 5 for each input, as required.

Transformations 3, 5, 6, 8, 9, and 11 are sometimes useful, and experience with the reduction technique will determine their application.

**EXAMPLE 7.9.** Let us reduce the block diagram (Fig. 7-10) to canonical form.

**Step 1:**

Transformations 8, 9, and 11 are sometimes useful, and experience with the reduction technique will determine their application.

**EXAMPLE 7.9.** Let us reduce the block diagram (Fig. 7-10) to canonical form.

![Block Diagram](Fig. 7-10)
Step 2:

An occasional requirement of block diagram reduction is the isolation of a particular block in a feedback or feedforward loop. This may be desirable to more easily examine the effect of a particular block on the overall system.

Isolation of a block generally may be accomplished by applying the same reduction steps to the system, but usually in a different order. Also, the block to be isolated cannot be combined with any others.

Rearranging Summing Points (Transformation 6) and Transformations 8, 9, and 11 are especially useful for isolating blocks.

**EXAMPLE 7.10.** Let us reduce the block diagram of Example 7.9, isolating block $H_1$.

Steps 1 and 2:
We do not apply Step 3 at this time, but go directly to Step 4, moving takeoff point 1 beyond block \( G_2 + G_3 \):

![Block Diagram](image)

We may now rearrange summing points 1 and 2 and combine the cascade blocks in the forward loop using Transformation 6, then Transformation 1:

![Block Diagram](image)

Step 3:

![Block Diagram](image)

Finally, we apply Transformation 5 to remove \( 1/(G_2 + G_3) \) from the feedback loop:

![Block Diagram](image)

Note that the same result could have been obtained after applying Step 2 by moving takeoff point 2 ahead of \( G_2 + G_3 \), instead of takeoff point 1 beyond \( G_2 + G_3 \). Block \( G_2 + G_3 \) has the same effect on the control ratio \( C/R \) whether it directly follows \( R \) or directly precedes \( C \).